The Homology of Loops on PD-complexes.

- M EMn.

- bool is to colculate Hx(SM) as an algebra.

- Generally SIM = TT X;

Hen $H_{\pi}(SM) \cong \otimes H_{\pi}(X_i)$ os mulles.

Bott-Sandson Thm: If x a a path-connected space the

 $H_{\pi}(\Omega \times X) \stackrel{\circ}{=} T(\hat{H}_{\pi}(X))$ (field coeffs)

and the suspension $X \stackrel{E}{\longrightarrow} \Omega \Sigma X$; hues the inducion $\hat{H}_{\chi}(X) \hookrightarrow T(\hat{H}_{\chi}(X))$,

More: If X is a suspension then this isomorphism is as Harf algebras.

Thm' Suppose I Whay coliberation EA frey by

where an has a croph hopy movered.

Let F: A - ary be the objoint of f.

Let R = Im (Fx). Then Folgebra

isomorphism

 $H_*(\Omega Y') \cong T(\widehat{H}_*(Y))/(R)$

when (R) is the 2-sided ideal grenated by R.

Remark: Ty is a suspension then this iso is as Harf ologhous.

Pti O asy ah ay'

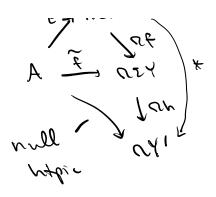
=> T(H*(41) (51h)* H*(UA,1) - olahua

As 5h has a right hopy inverse, get (5h) x is an epimorphism.

or The object f: A - OST is Which the composite

A E ard at ary.

Consider = 72A => 8h' of 28.



of solver or ty, and solver or algebra map, get a bodosisation

 $T(\widetilde{H}_{*}(Y)) \xrightarrow{(\Omega h)_{*}} H_{*}(\Omega Y')$ $T(\widetilde{H}_{*}(Y))$ $T(\widetilde{H}_{*}(Y))$

where b is an algebra map and an expinorphism.

Claim! b is an isonorphism.

noitesization a for a wood &

$$T(\widehat{H}_{*}(Y)) \xrightarrow{Q(K)_{*}} H_{*}(NY)$$

$$T(\widehat{H}_{*}(X))/(R)$$

The htpy cofibration EA ± 24 my 1

too the with a right htpy inverse so

by the enhanced version of ar c gives

a htpy libertion

24[7,8] 24 My Y3

Where & ERY is ERRY en EY

4 5 is a staget why means Conh.

Think of

(CR1E] +f)

S((QY'NEA)UZA) X SEY Rh NY!

- splite ner = nr' x ra(ar'realver)

 $(Asv[Ash]) = H_*(ny') \otimes H_*(n(ny') + H) = (I(y) + H)'T$

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Consider

 $H_{\star}(S(SY'NSN'YSN'YS))$

- Xx is an olg wap so it is determed by what bappens order precomposed with

(arianiva = allarira)

- NOE | = = = = Im fx

=> 00 X + 0 E + / A = 0.

- XOE/ MYRA is a Sandson product

(8/82 for some may 8'.

~ is o~

gan plo

= 0 or a, t, =0.

Therfore a. $X_{+} = 0$.

noitorization & and

T(A*(41) (B) T(A*(41) (B)

where c is an algebra nop, and an apinorphism.

D Combaco:

T(H* 471)/(B) - H*(BA, 1) POL(H*(A)) = H*(BA, 1)

- c, b are both algebra naps, b. the apinorphisms.
 - boc, cob are old mays, epinorphisms.
 - boc is a self-map of TLA+(41)/(R)

 cob " H*(841)
 - so cob, box are self-vaps of Linite type vidules that are epinocohisms = 1 thay are

J -M M - Q

where H+(0) = H+(5m x5n-m).

M J= ET tlen as sij has a right Whey movere get an algebra is:

H*(21) = L(H*(2m-112u-112))

Where R= Im Fx.

Note: \$: 50-3 - alsonsonvet)

Let == Im Fx - r is just a smale element

1~ 7(A*(5m-1-m-1))

=> H+(nn)=T(H+(5m-15n-m-7=)/(1).

- this is colled one-relator algebras.

Ex: More concrète, a special case of MEMn.

Let M = \$ (5m; * 5n-m;) where m; 7.2.4;

N'is a PD-complex, M + Mn.

gila hapy E

20-1 = 1 (2m; 12n-mi) = 1

where $f = \sum_{i=1}^{g} [1_{m_i}, 1_{m-m_i}]$

Sum of Whitehead probable.

 $\mathcal{H}^*(\mathcal{E}_{\nu-s}) \longrightarrow \mathcal{L}[\mathcal{H}^*(\mathcal{A}^{i-1} \mathcal{E}_{w^{i-1}} \mathcal{A}^{i-m_{i-1}})]$

= T(u,, ..., ud, v,, ..., vd)

luil = m; -1 , |vil = n-m;-1

The f*(1,-2) = [u, v, 7+ -- + Lud, vd].

Thus

 $H_{+}(S) = L(\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}, \alpha_{5})$ $= L(\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{5}, \alpha_{5}) / ((\alpha_{1}, \alpha_{2}, \alpha_{5}, \alpha_{5}, \alpha_{5}))$ olabora

If each m', n-m; >3, this is an iso or Harf algebras.