$V(x)$ が一定　$E > V$　波

$$\psi(x) = A \cdot \exp(ikx) + B \cdot \exp(-ikx)$$

進行波 $\longrightarrow$　　反射波 後退波 $\longleftarrow$

$$\Psi(x,t) = \underline{\psi(x)} \cdot \varphi(t)$$

$$\varphi(t) = \exp(-i\omega t)$$

$$= A\exp(i(kx-\omega t))$$

$$+ B\exp(+i(-kx-\omega t))$$

$\hbar\omega = E$ (粒子としての全エネルギー)

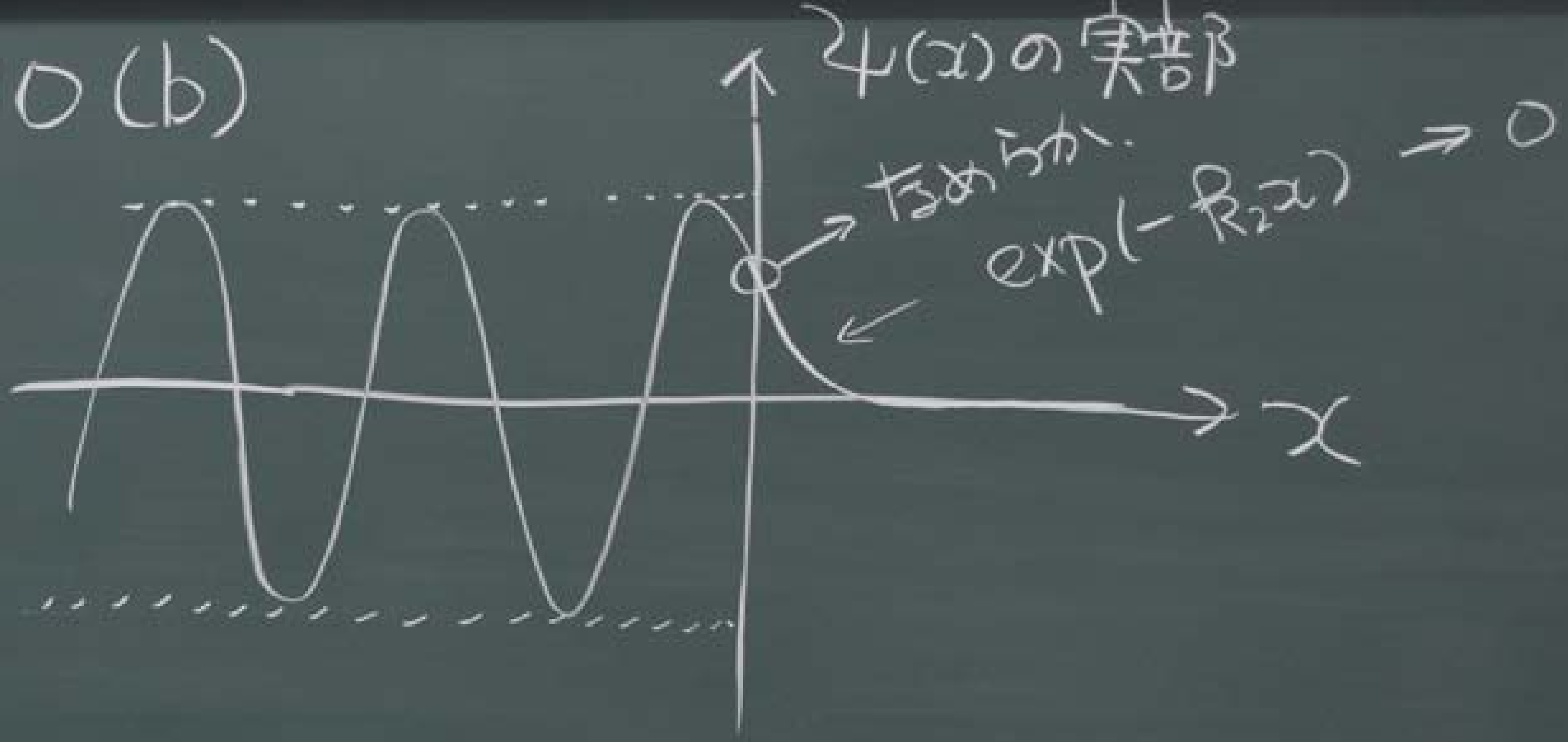
(1.80)

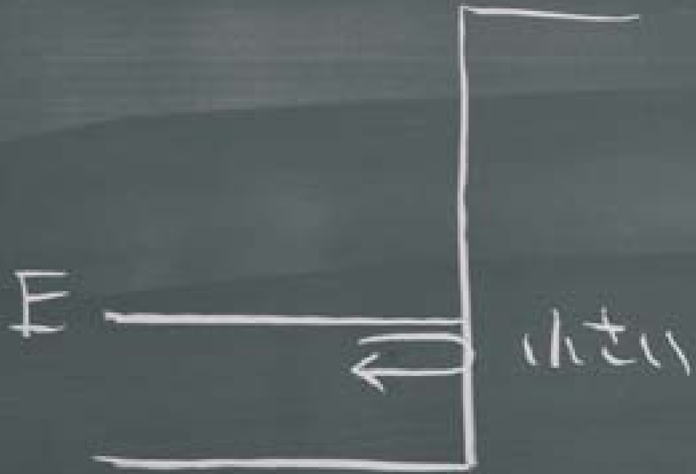
$$\Psi(x,t) = A \cdot \exp(i(kx - \omega t))$$

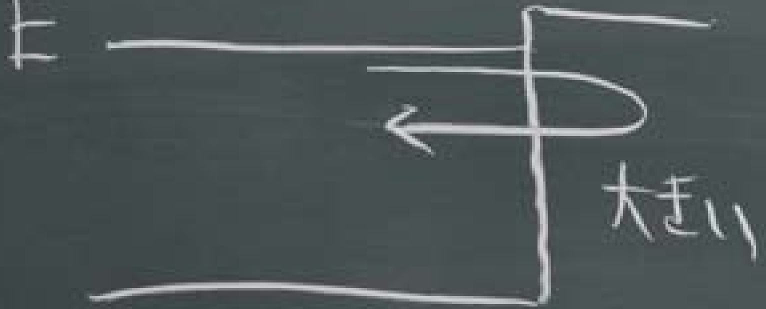
$$= A\left[\underbrace{\cos(kx - \omega t)}_{\text{実}} + i\,\underbrace{\sin(kx - \omega t)}_{\text{虚}}\right]$$

$$|\Psi(x,t)|^2 = A^2 = \text{空間的に一定}$$

図1.10(b)







$\exp(-k_2 x)$

k_2が小さい程
深く
しみ出す

$$k_2 = \sqrt{2m(V-E)}$$

$$\overset{A_2}{\left|\exp(-k_2 a)\right|^2}$$

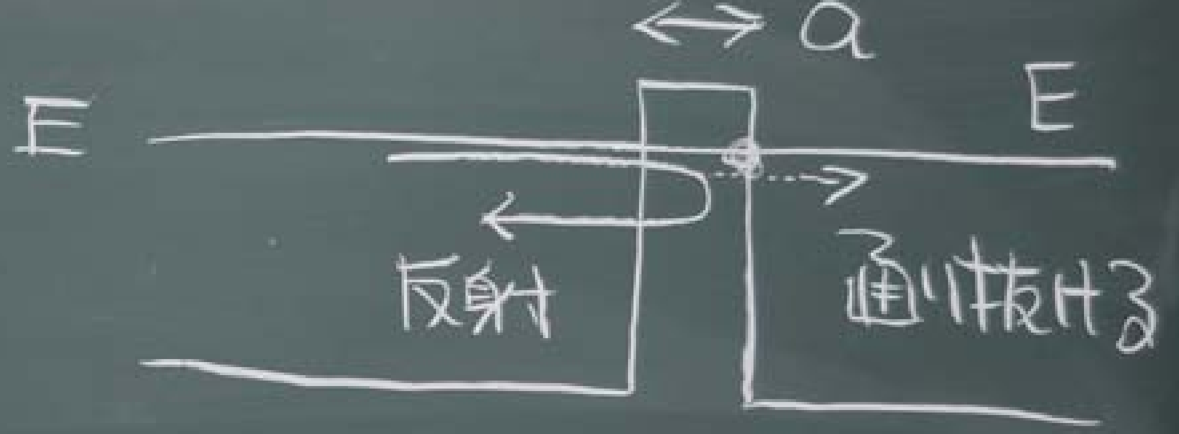
(1.102)

トンネル確率

$$w = A_2^2 \exp\left(-\frac{2\sqrt{2m(V-E)}\,a}{\hbar}\right)$$

a が小さい程 大

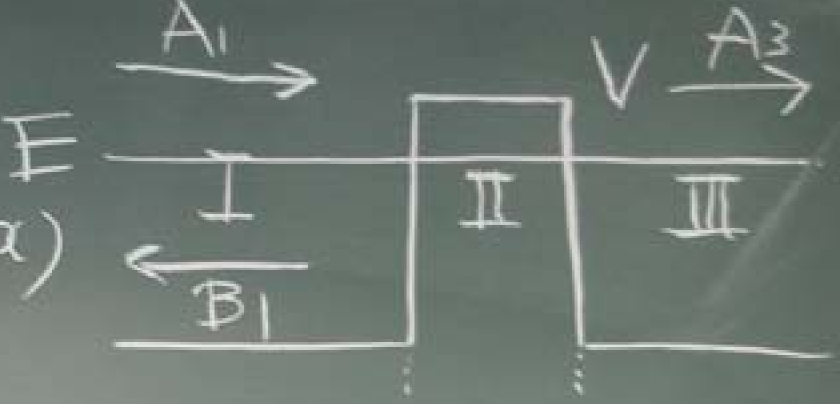
V が小さい程 大



量子力学的トンネル効果

通り抜けた後もエネルギー同じ

E<Vの場合を考える.



(I) $\psi_{\mathrm{I}}(x) = A_1 \exp(ik_1 x) + B_1 \exp(-ik_1 x)$

(II) $\psi_{\mathrm{II}}(x) = A_2 \exp(k_2 x) + \underline{B_2 \exp(-k_2 x)}$

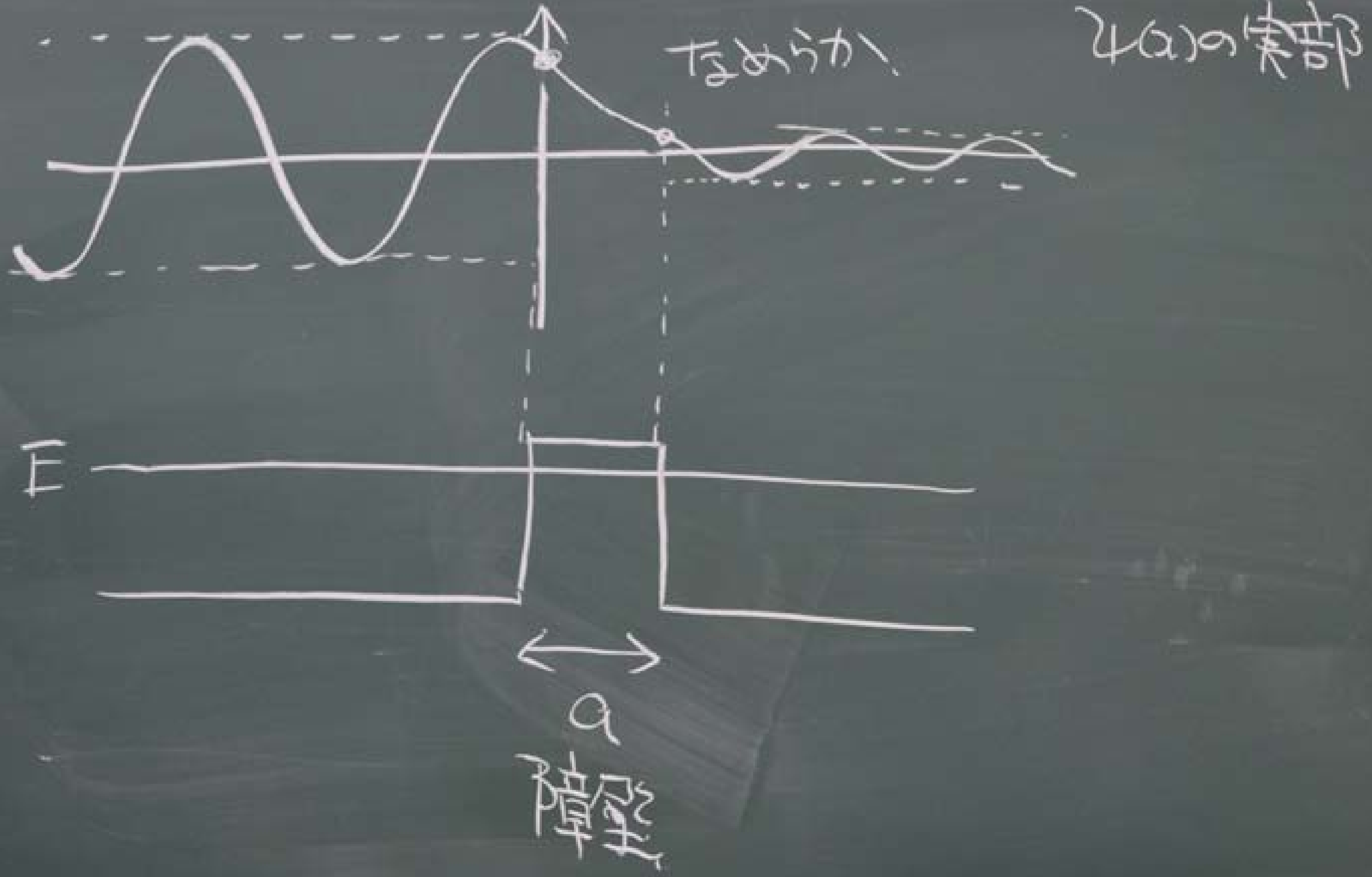
残す.

(III) $\psi_{\mathrm{III}}(x) = A_3 \exp(ik_3 x)$

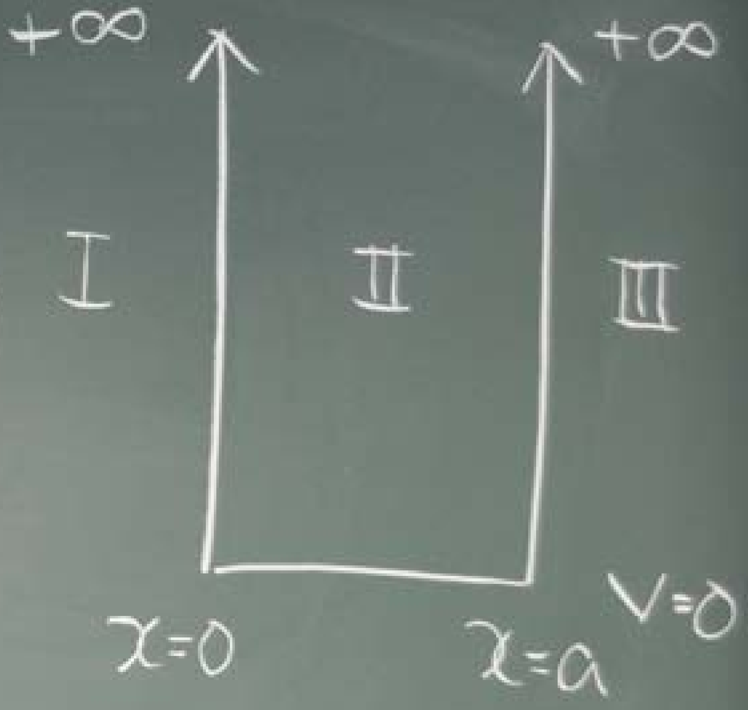
$$\begin{cases} \psi_{\mathrm{I}}(0) = \psi_{\mathrm{II}}(0) \\ \psi'_{\mathrm{I}}(0) = \psi'_{\mathrm{II}}(0) \end{cases} \quad \begin{cases} \psi_{\mathrm{II}}(a) = \psi_{\mathrm{III}}(a) \\ \psi'_{\mathrm{II}}(a) = \psi'_{\mathrm{III}}(a) \end{cases} \rightarrow (1.112)$$

$$T = \frac{|A_3|^2}{|A_1|^2} = \frac{4k_1^2 k_2^2}{(k_1^2 - k_2^2)^2 \sinh^2 k_2 a + (2i k_1 k_2 \cosh k_2 a)^2}$$

$$k_2 a \gg 1 \quad \sim \frac{4}{e^{2k_2 a} + 4} \sim 4 \cdot \exp(-2k_2 a)$$


ψ(a)の実部
なめらか
E
a
障壁

$$\begin{cases} \psi_{\mathrm{I}}(x)=0 \\ \psi_{\mathrm{II}}(x)=A_1\exp(ikx)+B_1\exp(-ikx) \\ \psi_{\mathrm{III}}(x)=0 \end{cases} \qquad (1.119)$$



$V=\infty$が出てくる場合 微分不連続でよい。

$$\begin{cases} \psi_{\mathrm{I}}(0) = \psi_{\mathrm{II}}(0) \\ \psi_{\mathrm{II}}(a) = \psi_{\mathrm{III}}(a) \end{cases}$$

$$\begin{cases} A_1 + B_1 = 0 \\ A_1 \exp(ik_2 a) + B_2 \exp(-ik_2 a) = 0 \end{cases}$$

$A_1 = B_1 = 0$ 自明な解 以外の解をもつ.

$$\begin{vmatrix} 1 & 1 \\ \exp(ik_2 a) & \exp(-ik_2 a) \end{vmatrix} = 0$$

$\rightarrow$

$$\exp(2ik_2 a) = 1$$

この時のみ

意味のある解

$\rightarrow$ 量子条件

$$\exp(2ik_2a) = 1$$

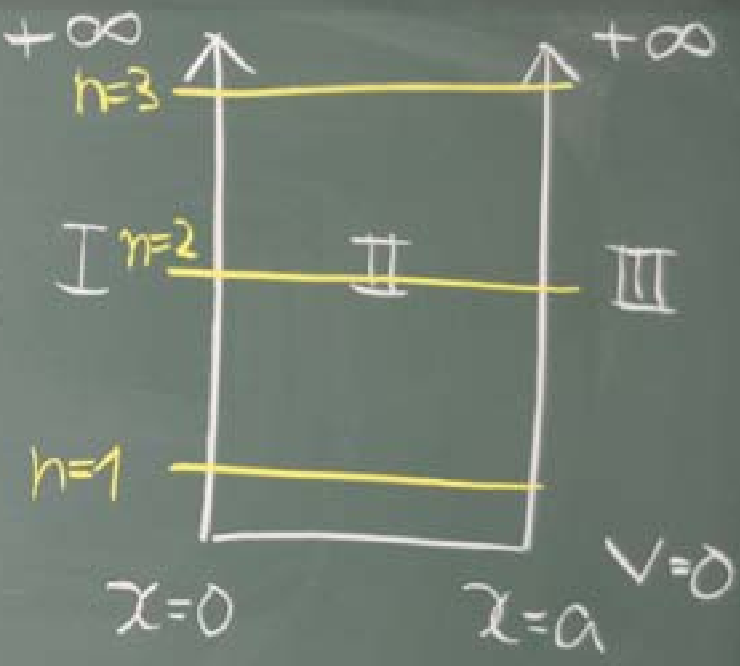
$$2k_2a = 2n\pi \quad (n\text{:整数})$$

$$k_2a = n\pi.$$

$$k_2 = \sqrt{2mE}$$

$$E_n = \frac{n^2h^2}{8ma^2}$$

$$
\begin{cases}
\psi_{\mathrm{I}}(x) = 0 \\
\psi_{\mathrm{II}}(x) = A_1 \exp(ikx) + B_1 \exp(-ikx) \\
\psi_{\mathrm{III}}(x) = 0 \qquad (1.119)
\end{cases}
$$



V = ∞ が出てくる場合 微分不連続でよい.

$$\exp(2ik_2a) = 1$$

$$2k_2a = 2n\pi \quad (n\text{: 整数})$$

$$k_2a = n\pi.$$

$$k_2 = \sqrt{2mE}$$

$$E_n = \frac{n^2h^2}{8ma^2}$$

① 値が飛び飛び

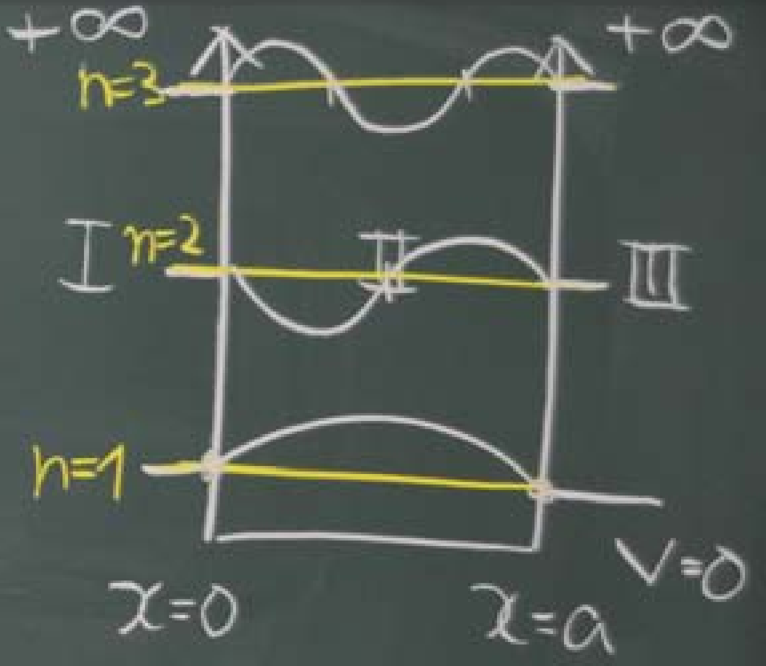
② 最低エネルギーが 0 で無い

$$\psi_{II}(x) = A \cdot \left[\exp(ik_2 x) - \exp(-ik_2 x) \right]$$

$$= \underbrace{2iA}_{\text{定数}} \cdot \sin(k_2 x)$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

$$
\begin{cases}
\psi_{\mathrm{I}}(x) = 0 \\
\psi_{\mathrm{II}}(x) = A_1 \exp(ikx) + B_1 \exp(-ikx) \\
\psi_{\mathrm{III}}(x) = 0 \qquad (1.119)
\end{cases}
$$



$V=\infty$ が出てくる場合　微分不連続でよい。

$$\psi_{\mathrm{II}}(x) = A \cdot \left[\exp(ik_2 x) - \exp(-ik_2 x) \right]$$

$$= \underbrace{2iA}_{\text{定数}} \cdot \sin(k_2 x)$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1 \quad = \int_0^a |\psi(x)|^2$$

(1.125)

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \times \underset{\sim\sim\sim}{e^{-i\omega t}}$$

$$w_n(x) = |\psi_n(x)|^2 = \frac{2}{a} \sin^2\left(\frac{n\pi x}{a}\right)$$

図1.13　波動関数ではなく確率密度の図