

# A few problems on functional self-organization in brain

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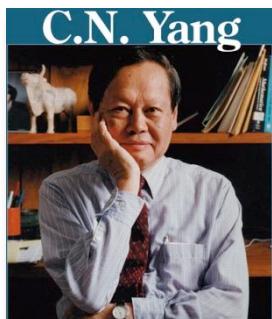
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YITP What's Life 2007.10

# APCTP

<http://www.apctp.org>

Asia Pacific Center for Theoretical Physics



**Founding President**



**Second President**



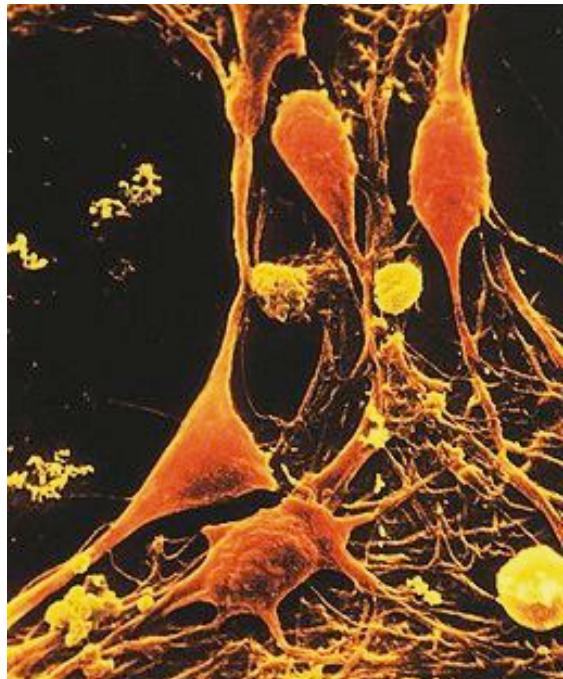
**Peter Fulde**

**Third President**

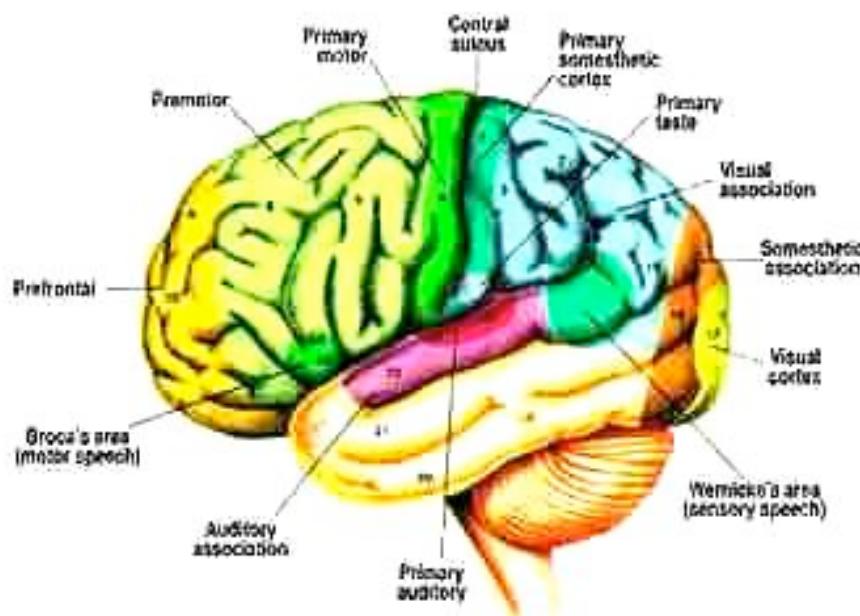
- Founded in 1996, located in Pohang, Korea
- 12 member countries, coop. with YITP, KEK, RIKEN, ISSP,...
- International cooperation, academic activities, young sci. training
- 3 M \$/year, 40 activities, ~2,000 visitors

# Outline

1. Introduction
2. Functional Self-organization in Cortical Development
3. Functional network formation in a STDP Neural Network Model
4. Functional Pathways in EEGs
5. Conclusion



# Brain



- Neuron : Fundamental building blocks
  - slow (~60Hz), nonlinear, complex response

- Massively interconnected
  - 1,000,000,000,000 neurons
  - 1,000,000,000,000,000 synapse

Circuits: layers, feedback loops, ...  
“networks of networks”

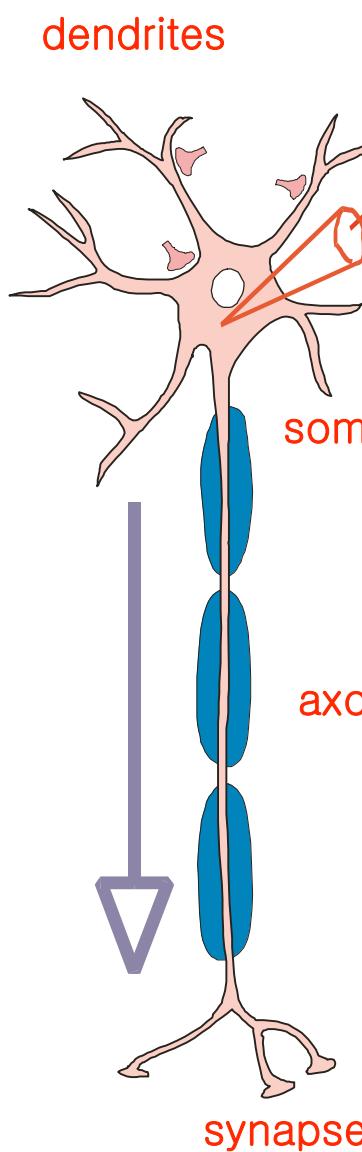
- Highly complex & correlated responses
  - ⇒ Sensory information processing, movement coordination, learning, memory, cognition...

- Activity dependent neural plasticity
  - e.g. Learning and memory

Q: Structure <-> Function?

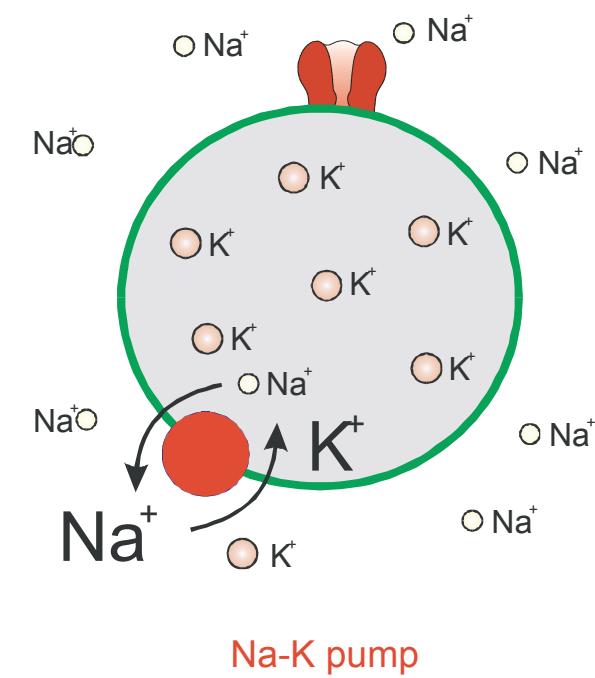
Functional self-organization

# Neural signal generation –Electrochemical



Neuron computes by dynamics of membrane potentials

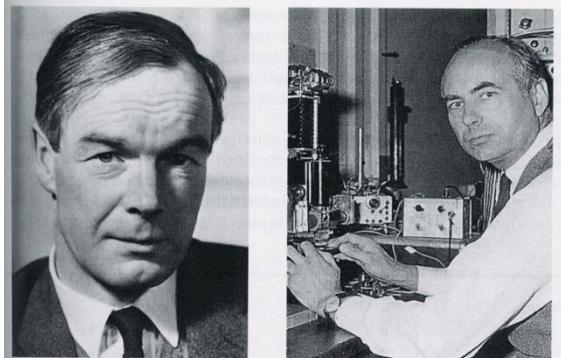
Complex electro–biochemical processes underlie neural signal generation



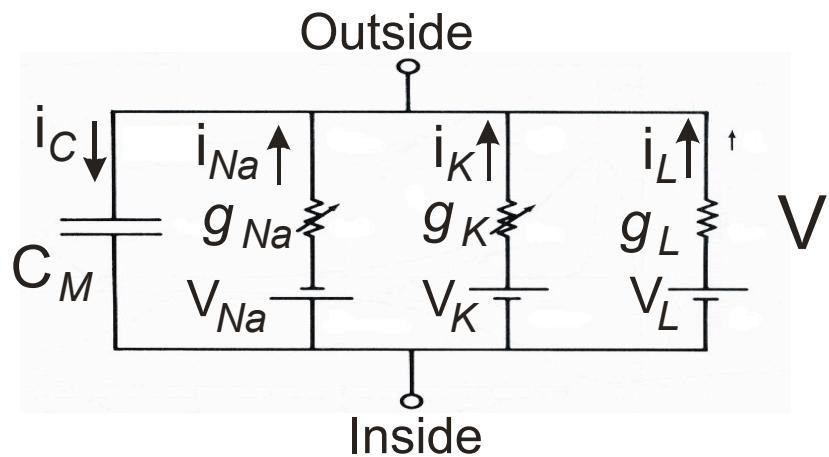
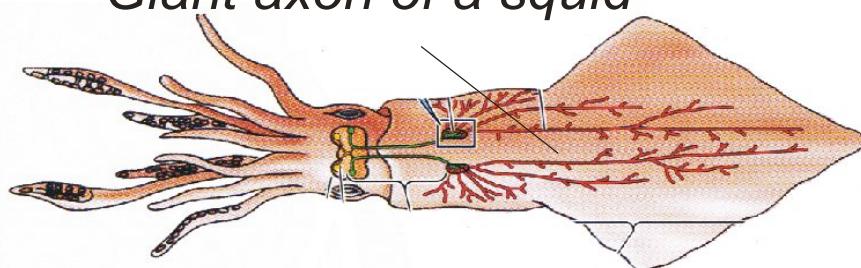
- Ion conductances => currents
- Nonlinear voltage gating

# Modeling a neuron – Hodgkin–Huxley

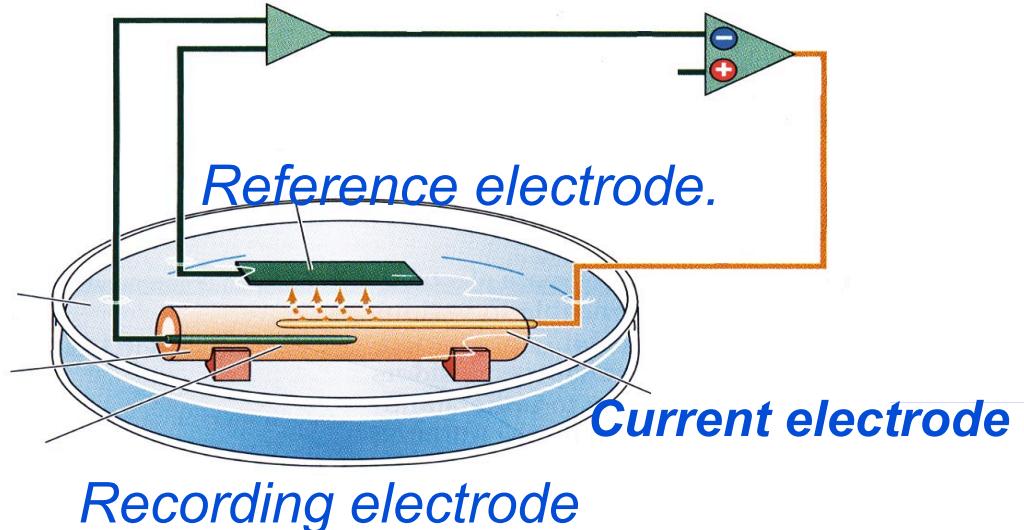
Hodgkin-Huxley, 1963



Giant axon of a squid



Voltage clamp experiment, 1952



Recording electrode

Single-compartmental circuit model

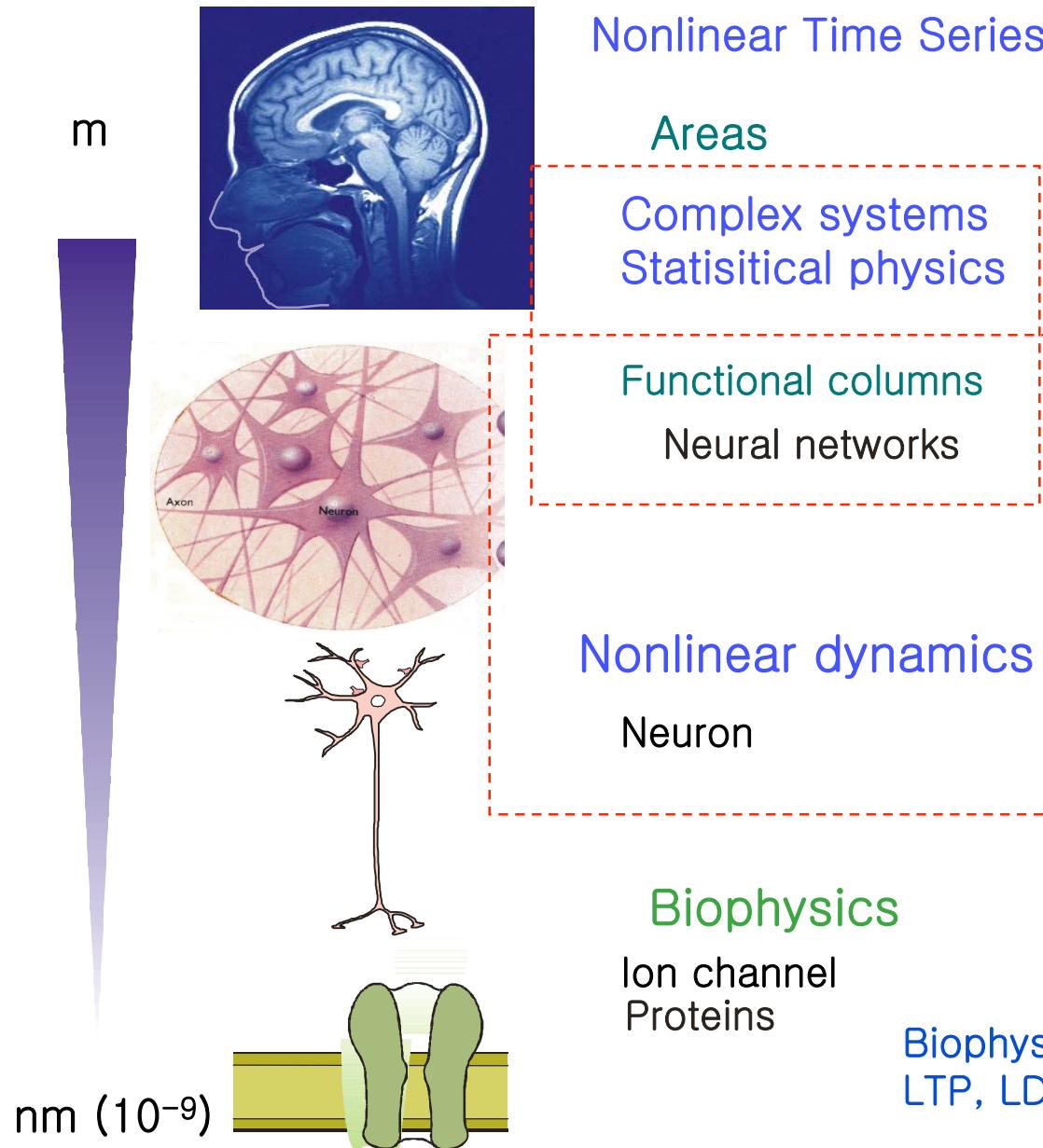
⇒ H-H neuron “Circuits” – 4-d ODE  
& variations of conduction-based models

Connection geometry : Global, local, sparse  
Synaptic interactions : STDP,  $\alpha$ , ...

External stimulus : dc, ac, noise

⇒ Dynamical System Models  
for Nervous Dynamics

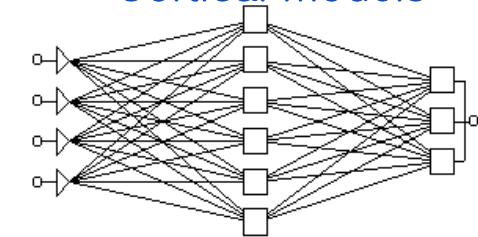
# Studying self-organization in a brain hierarchy



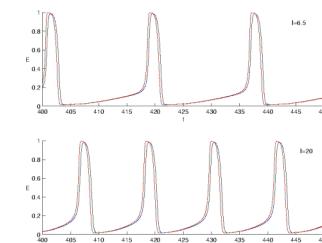
EEG/MEG pathways



Cortical models



Neural network models

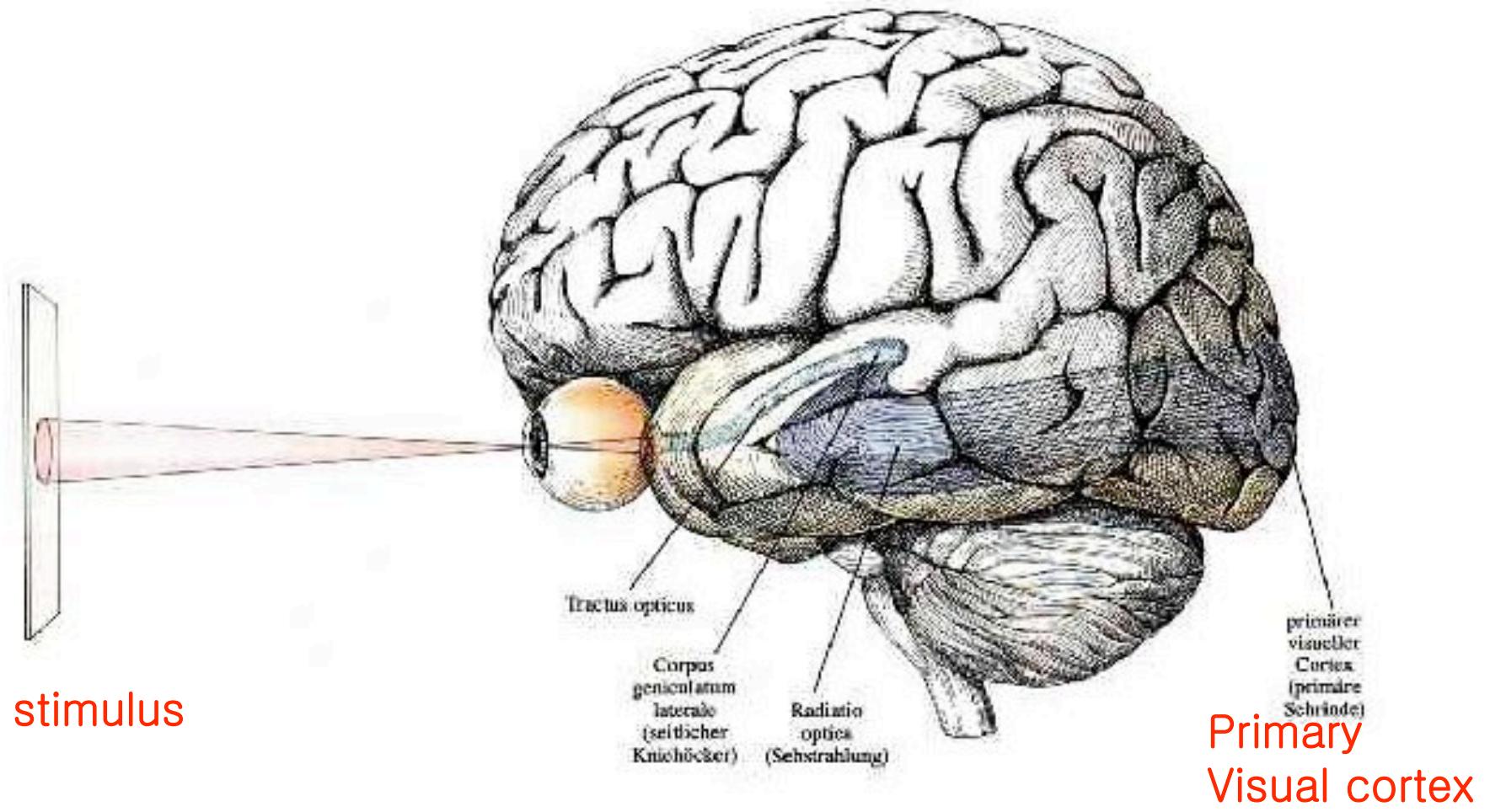


Nonlinear dynamic models



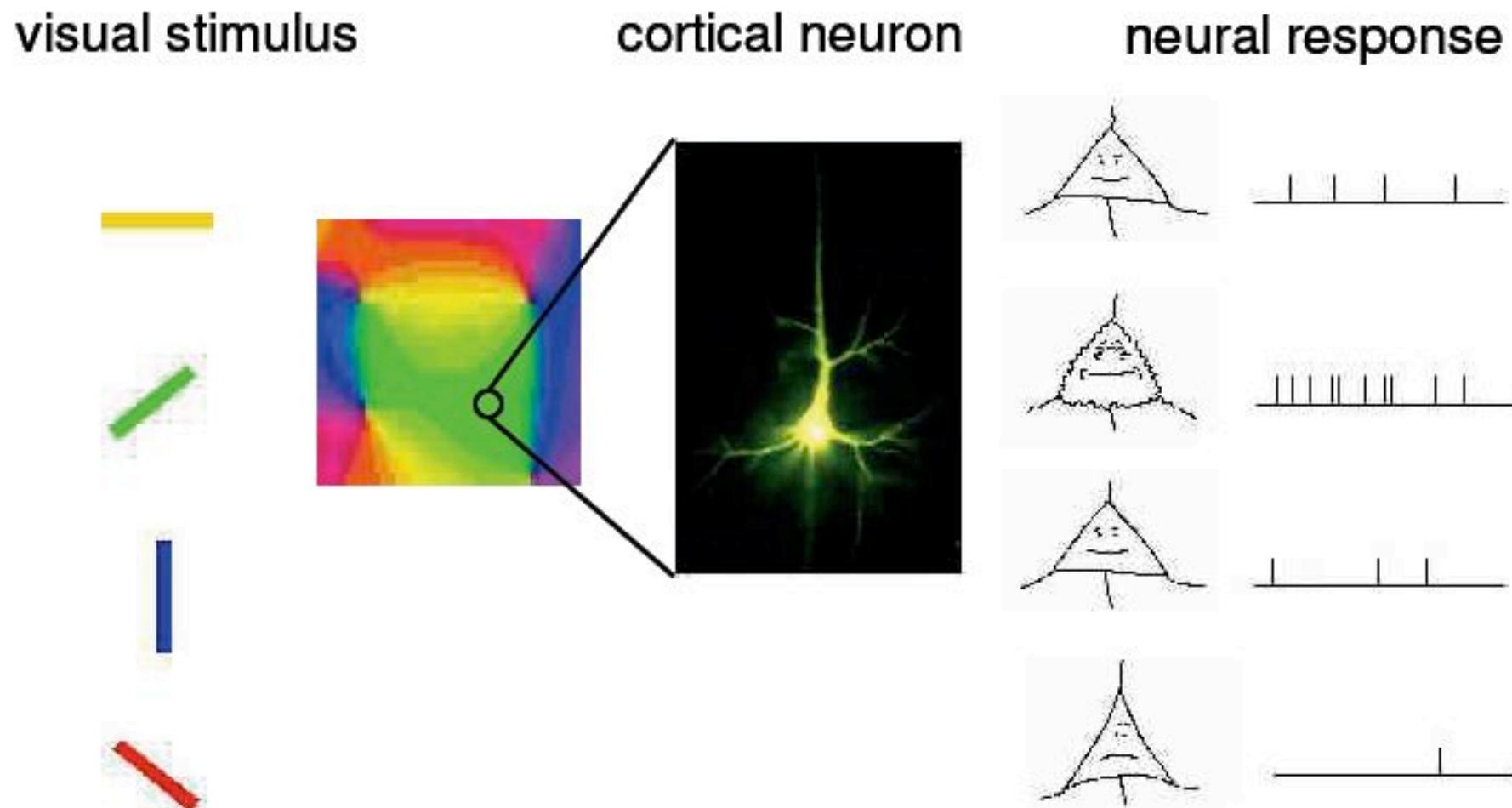
Biophysical models  
LTP, LDP, STDP

# Functional Self-organization in visual maps



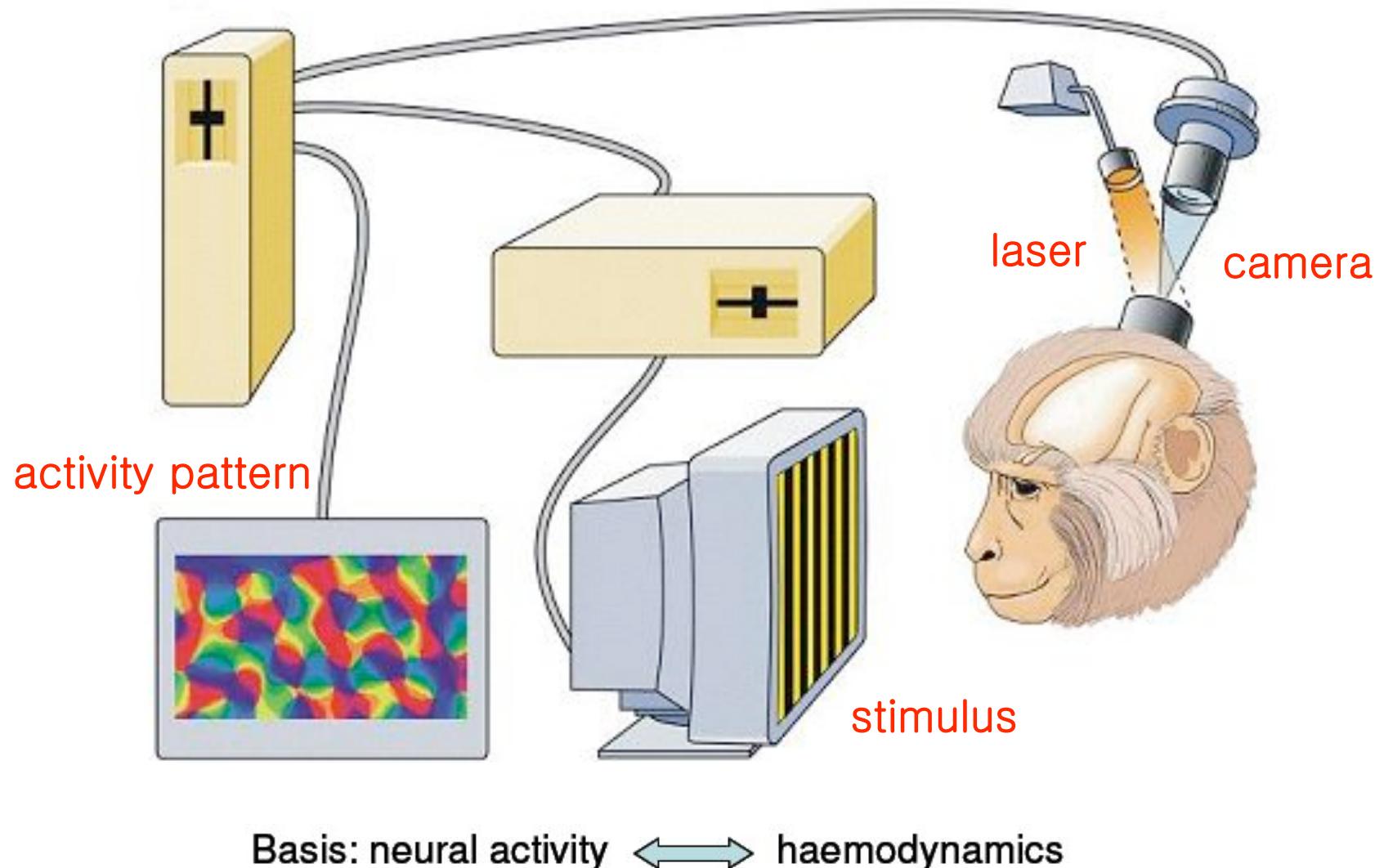
Hubel (1989)

# Orientation preference



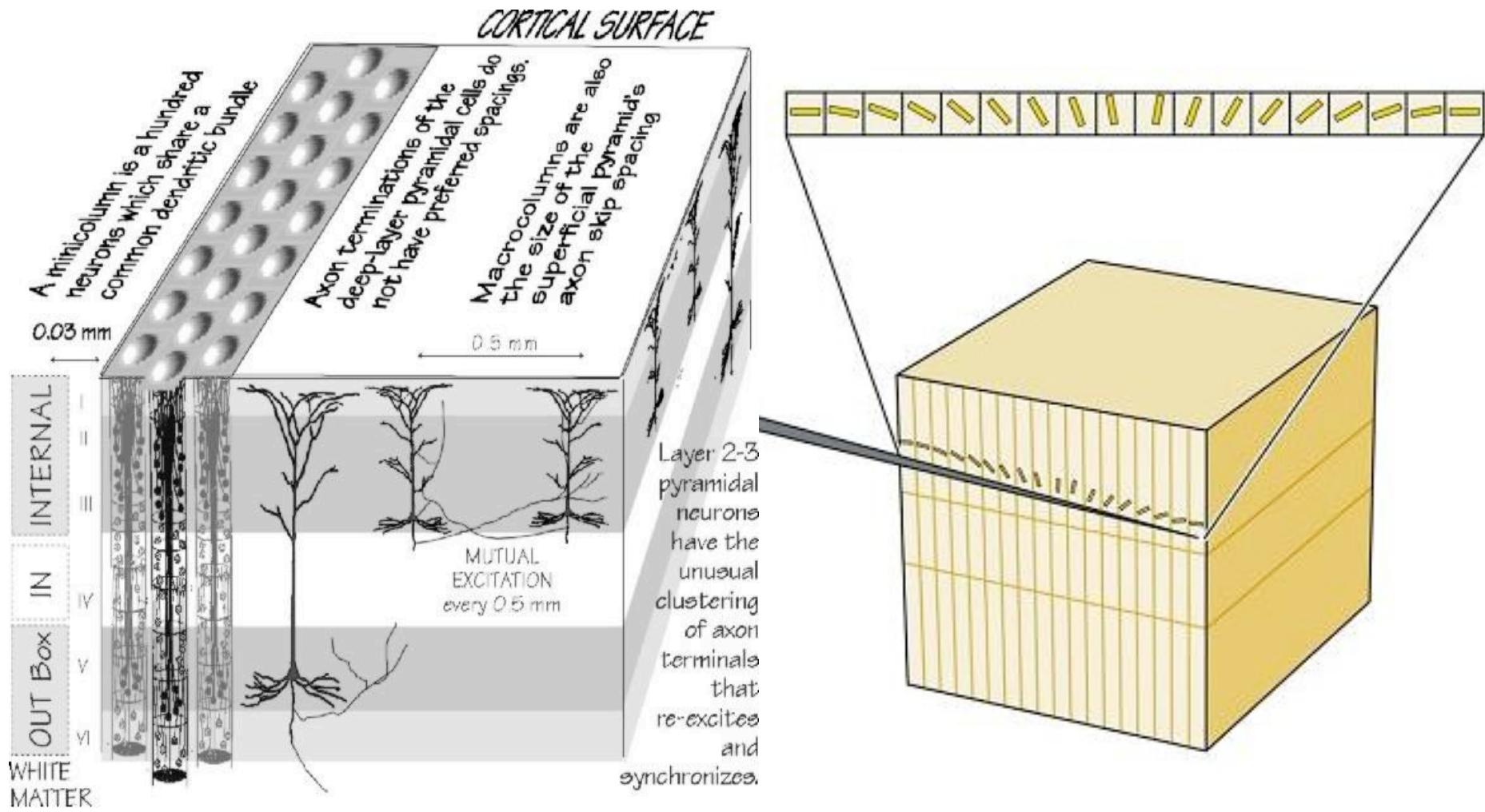
Hubel, Wiesel, J. Physiol. (1962)

# Optical imaging technique

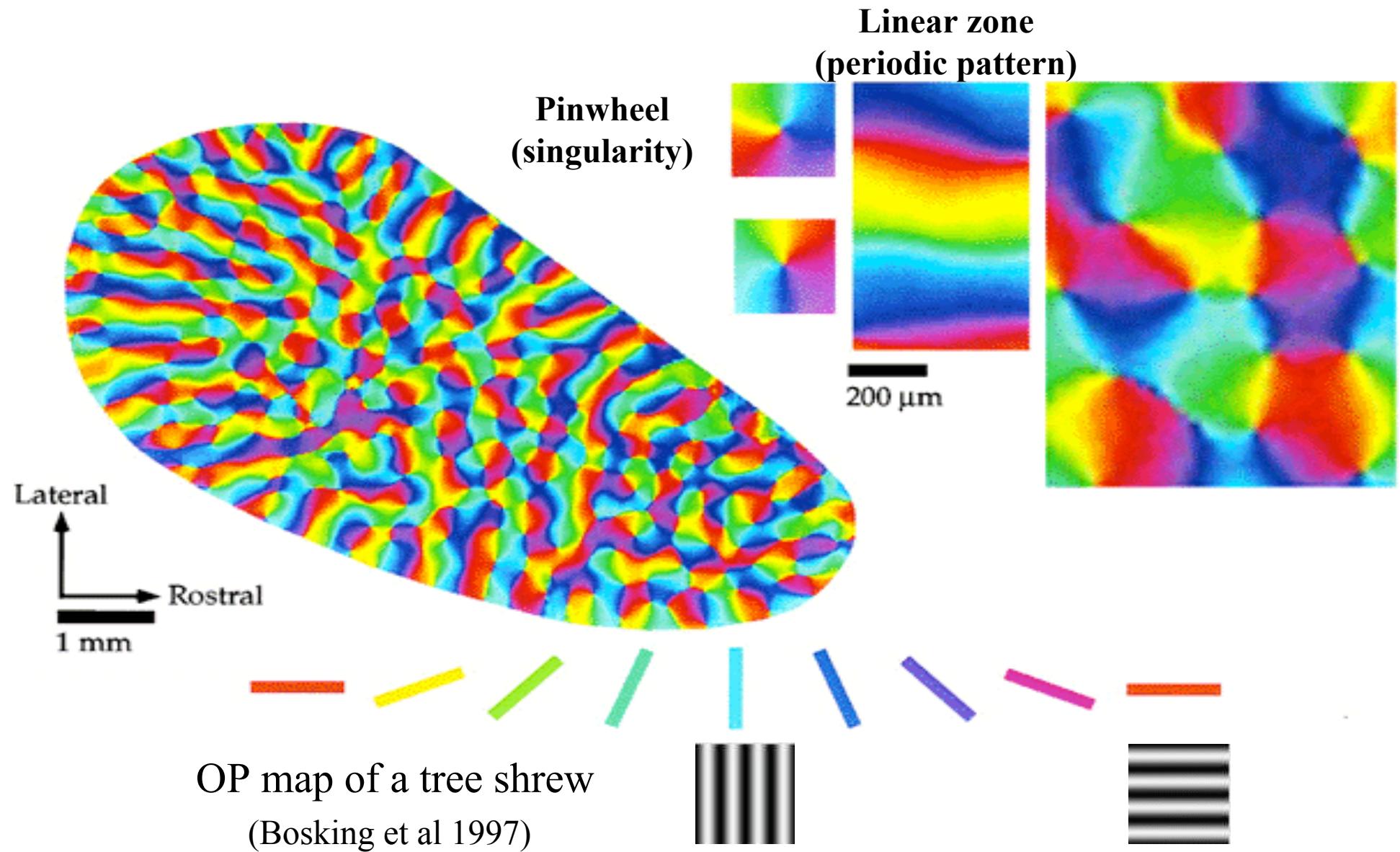


Bonhoeffer & Grinvald, J Neurosci (1993)

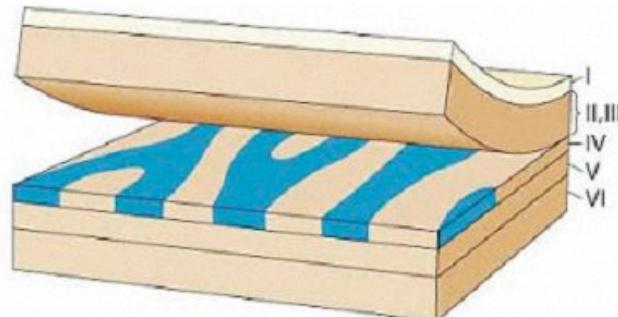
# Orientation preference columns



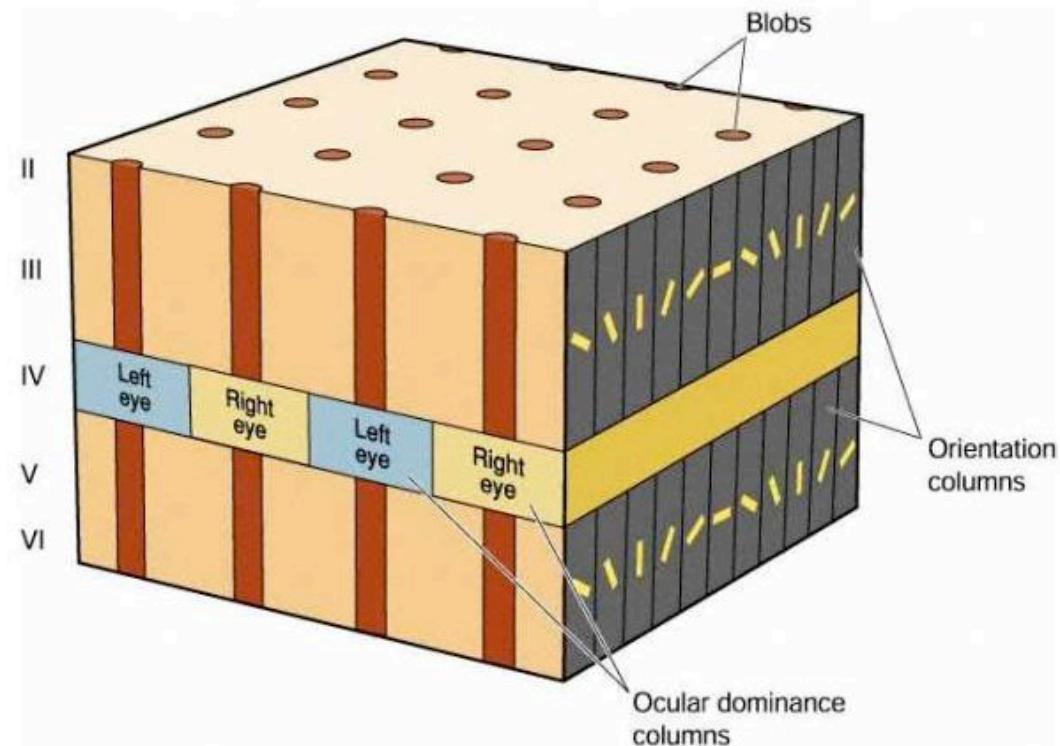
# Orientation preference (OP) map



# Ocular dominance (OD) map in V1

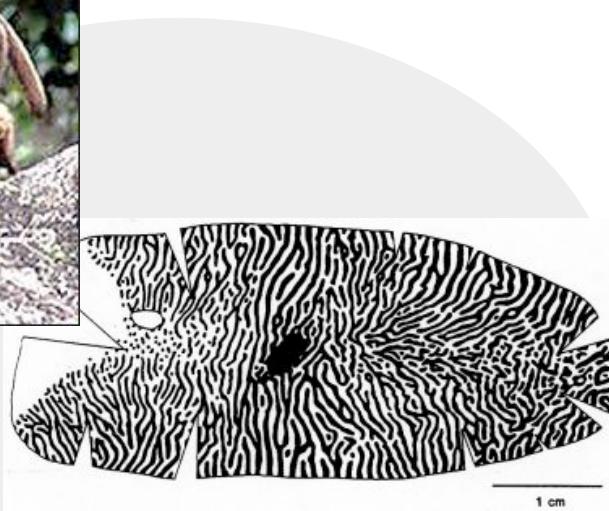


Ocular (left or right eye)  
dominance column



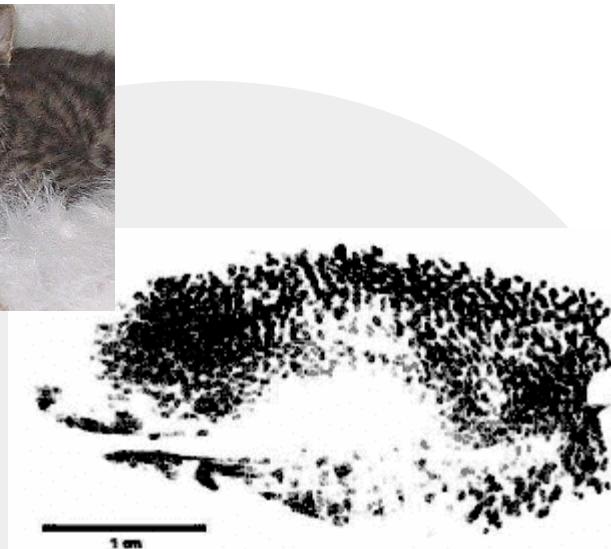
The structure of the  
primary visual cortex

# Different visual map types in mammals



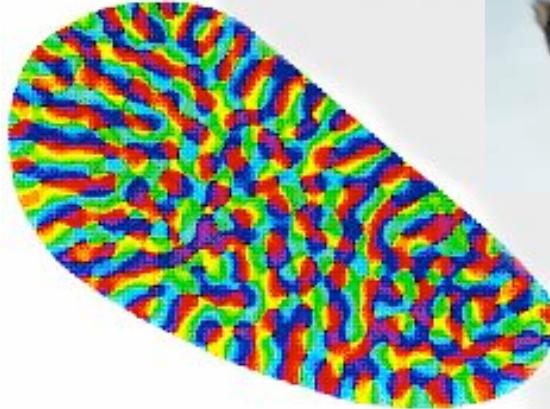
**Macaque Monkey**

- Stripe patterns in OD maps
- Strong OD segregation



**Cat**

- island patterns in OD maps
- intermediate OD segregation

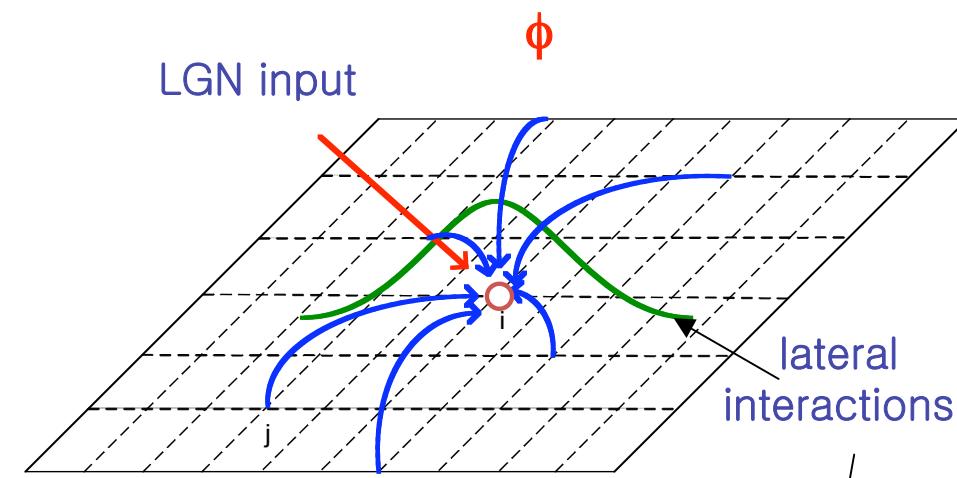


**Tree Shrew**

- Weak OD segregation (no OD map)
- Strong orientation selectivity

# A Biophysical model for visual map formation

2-dim layer of visual columns → Orientation preference



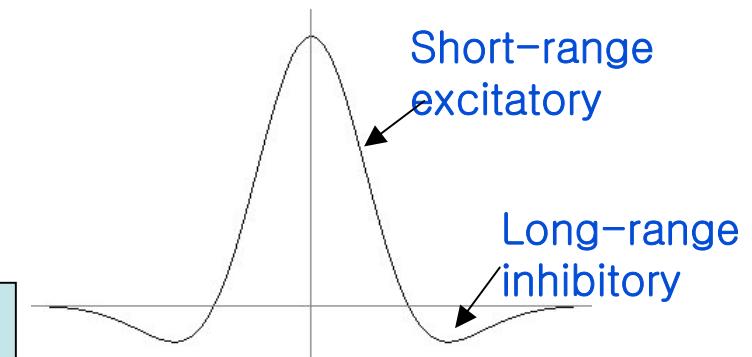
$$\frac{\partial \phi_i}{\partial t} = -\frac{\partial H}{\partial \phi_i} = -2\varepsilon \sum_j I(\vec{r}_i, \vec{r}_j) \sin(2\phi_i - 2\phi_j) - 2\mu B_i \sin(2\phi_i - 2\phi'_i)$$

LGN input

Coarse-grained picture at the functional column level

Neuron → functional column  
→ area maps

Distance dependent interaction



$I(r)$  : Mexican hat type

$$I(r) = \frac{1}{2} \left( 1 - k \frac{r^2}{\sigma^2} \right) \exp(-r^2 / 2\sigma^2)$$

M.W. Cho & S. Kim, PRL, 2004

# Analogy with a Spin Hamiltonian

**Orientation Preference Ocular Dominance**

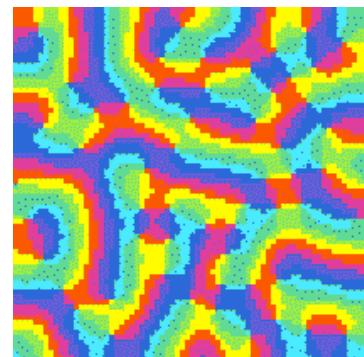
$$\mathbf{S}_i = (\cos 2\phi, \sin 2\phi)$$

$$\mathbf{S}_i = +1 \text{ or } -1$$

Maxican hat  
Distance dependent

$$H = - \sum_{i,j} J(\vec{r}_i, \vec{r}_j) \mathbf{S}_i \cdot \mathbf{S}_j - \sum_i h_i \cdot \mathbf{S}_i$$

X-Y model like



OP



OD

# Time evolution of visual map over time

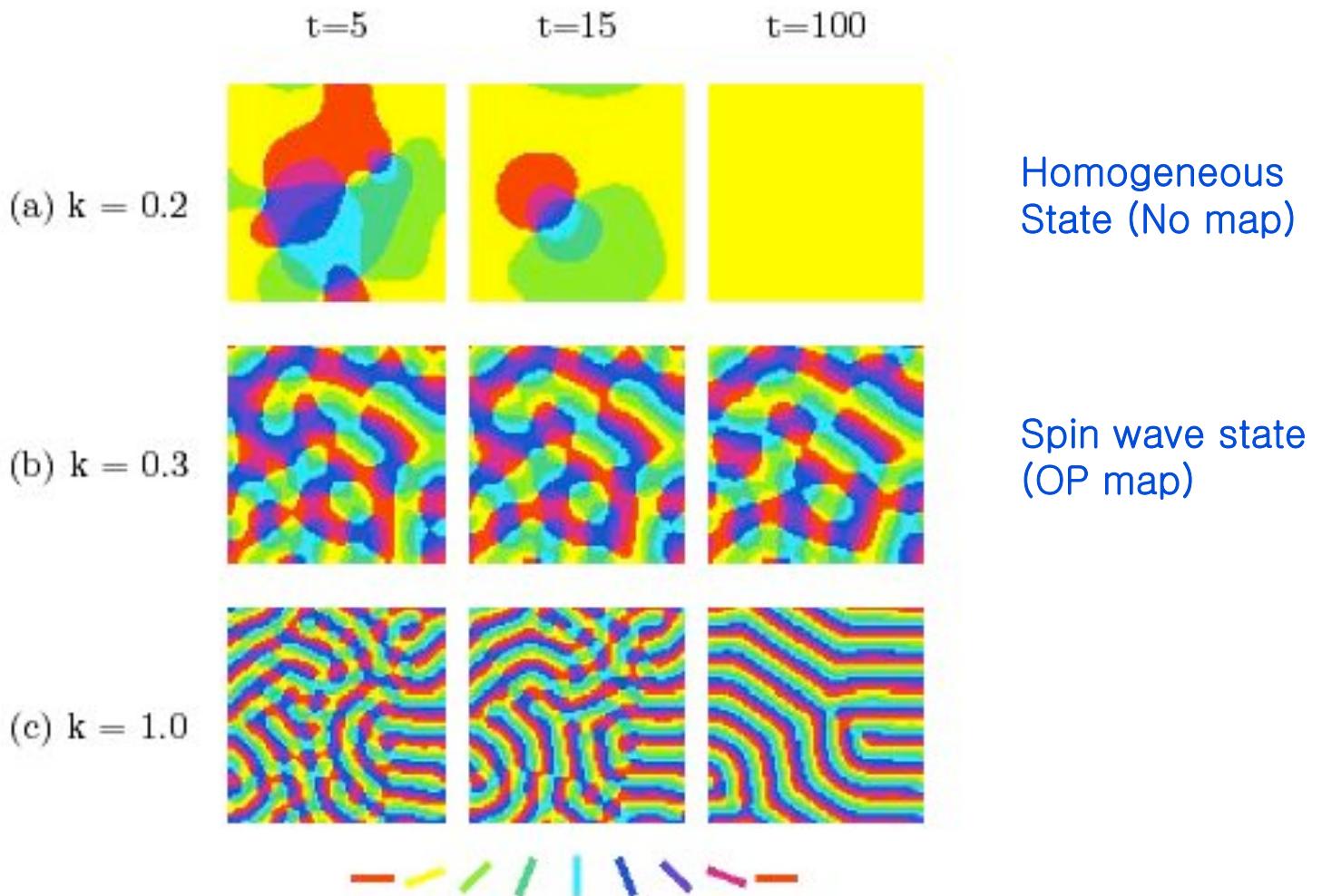


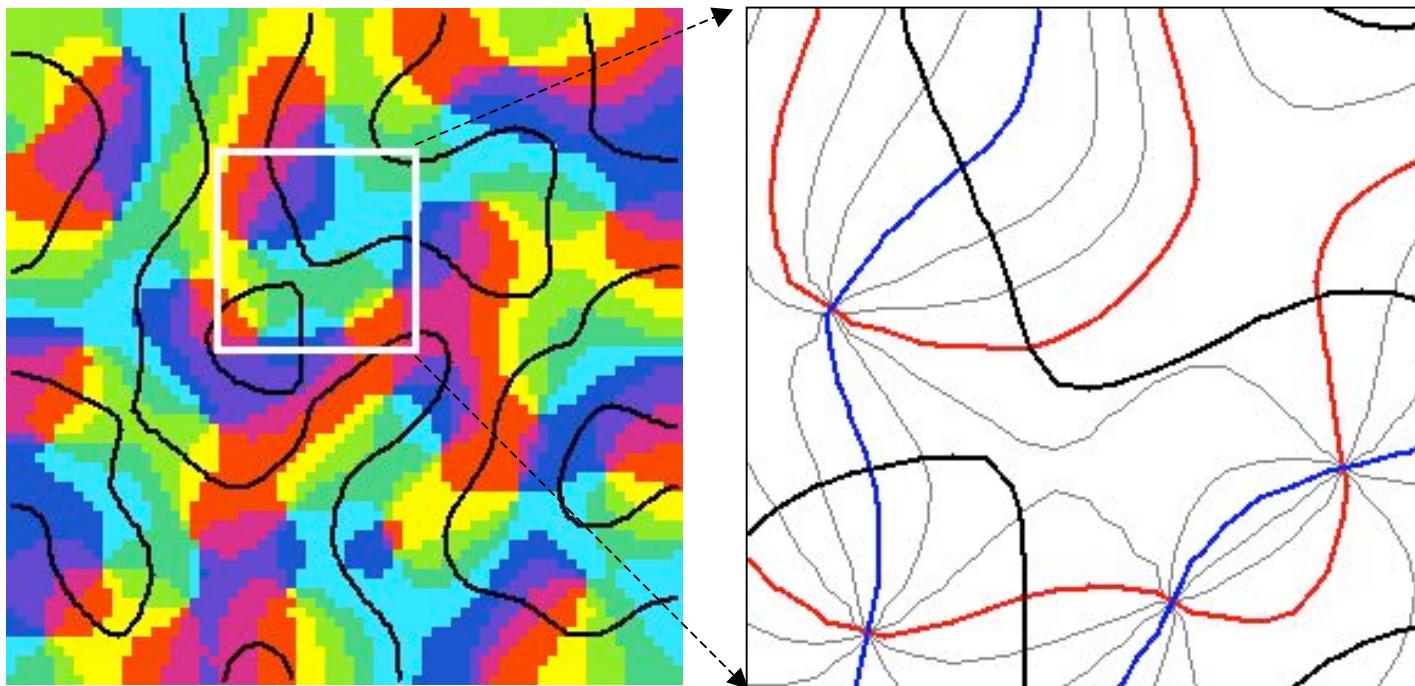
FIG. 2: Simulation results of the OP map using Eq. (1). Maps are generated with  $\sigma^2 = 6$ ,  $\varepsilon = 10^{-3}$ ,  $\mu = 0$  (zero temperature), periodic boundary condition and an initially random state in  $70 \times 70$  lattice.

# OP & OD correlations in a spin model

$$H = - \sum_{i,j} \left\{ J_{OP}(r_{ij})(S_i^x S_j^x + S_i^y S_j^y) + J_{OD}(r_{ij})S_i^z S_j^z \right\}$$

Red :  $S_x=0$  (OP,  $\phi=0$  or  $\pi/2$ )  
Blue :  $S_y=0$  (OP,  $\phi= \pi/4$  or  $3\pi/4$ )

Black :  $S_z=0$  (OD, +/- boundary)



OP/OD correlation  $\leq$  synaptic input normalization & competition

Theoretical predictions are consistent with experiments.

# Analogy with vortices in magnetism

Generalized Model of OP+OD

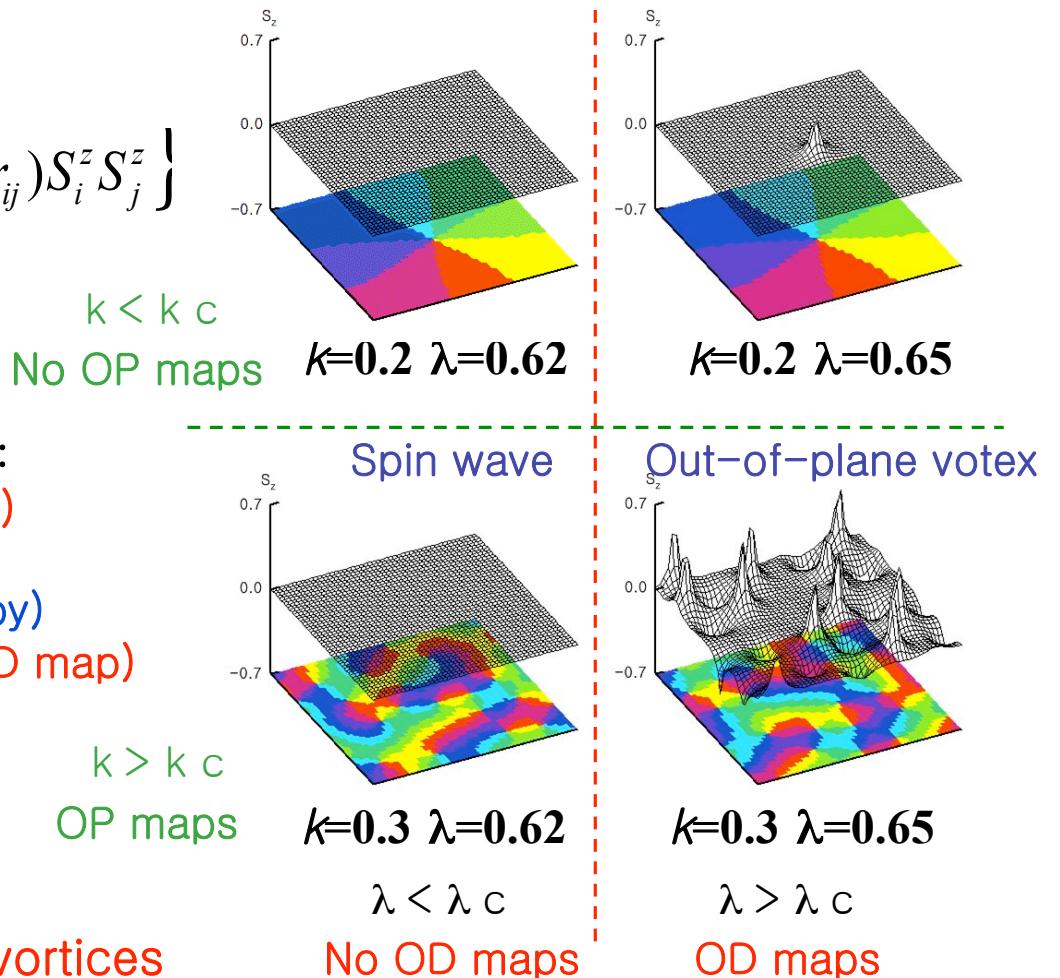
$$H = - \sum_{i,j} \left\{ J_{OP}(r_{ij})(S_i^x S_j^x + S_i^y S_j^y) + J_{OD}(r_{ij})S_i^z S_j^z \right\}$$

Exist bifurcations

- Bifurcation over  $k$  (inhibitory strength) : homogeneous  $\Leftrightarrow$  spin waves (OP map)
- Bifurcation over  $\lambda \propto J_{OD}/J_{OP}$  (anisotropy)  
in-plane  $\Leftrightarrow$  out-of-plane vortices (OD map)

OP pinwheels  $\Leftrightarrow$  in-plane vortices

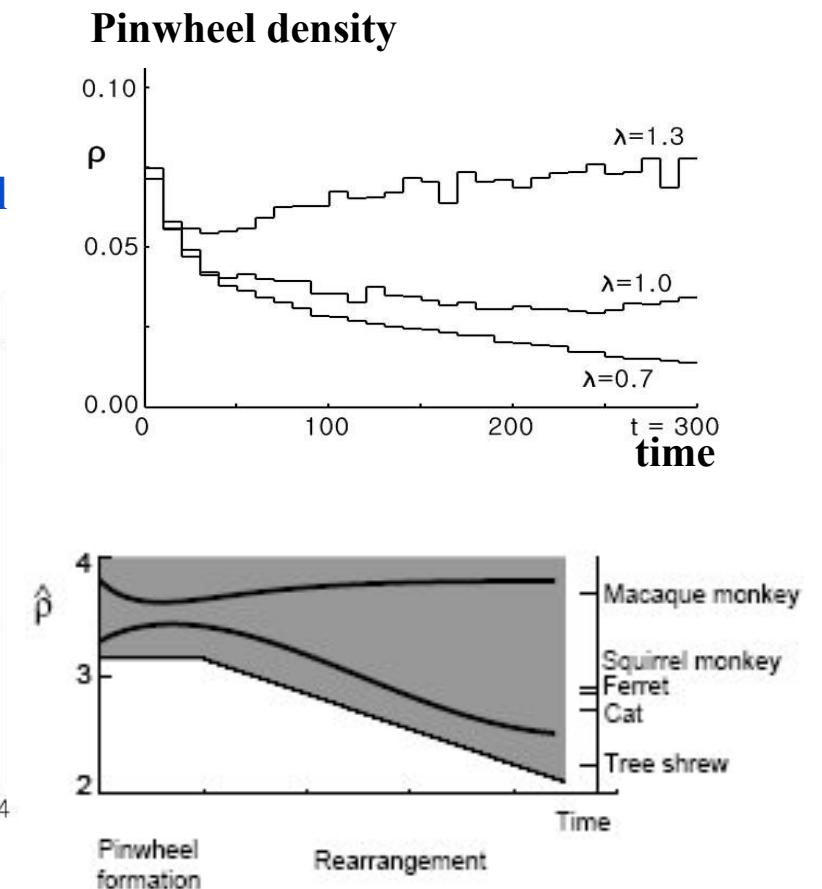
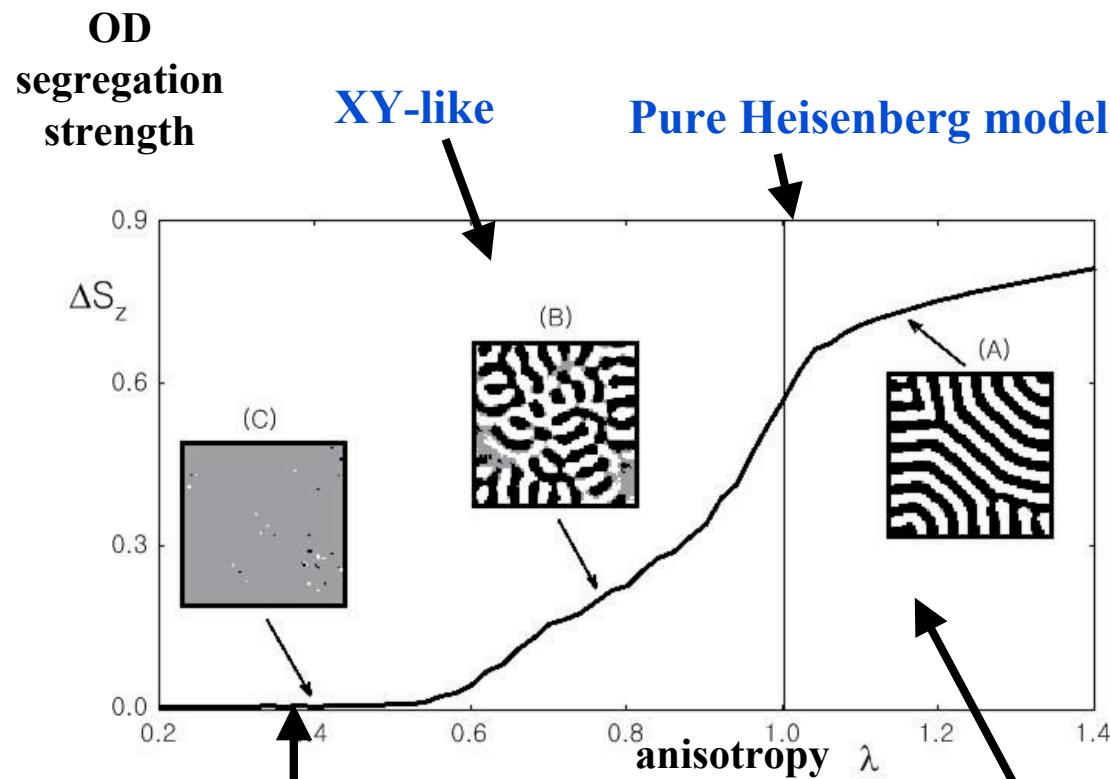
OD beaded bands  $\Leftrightarrow$  out-of-plane vortices



\* Anisotropic Heisenberg model  $H = -K \sum_{\langle i,j \rangle} \left\{ S_i^x S_j^x + S_i^y S_j^y + \lambda S_i^z S_j^z \right\}$

M.W. Cho & S. Kim, PRB, 2004

# Different OD map formation in a spin model

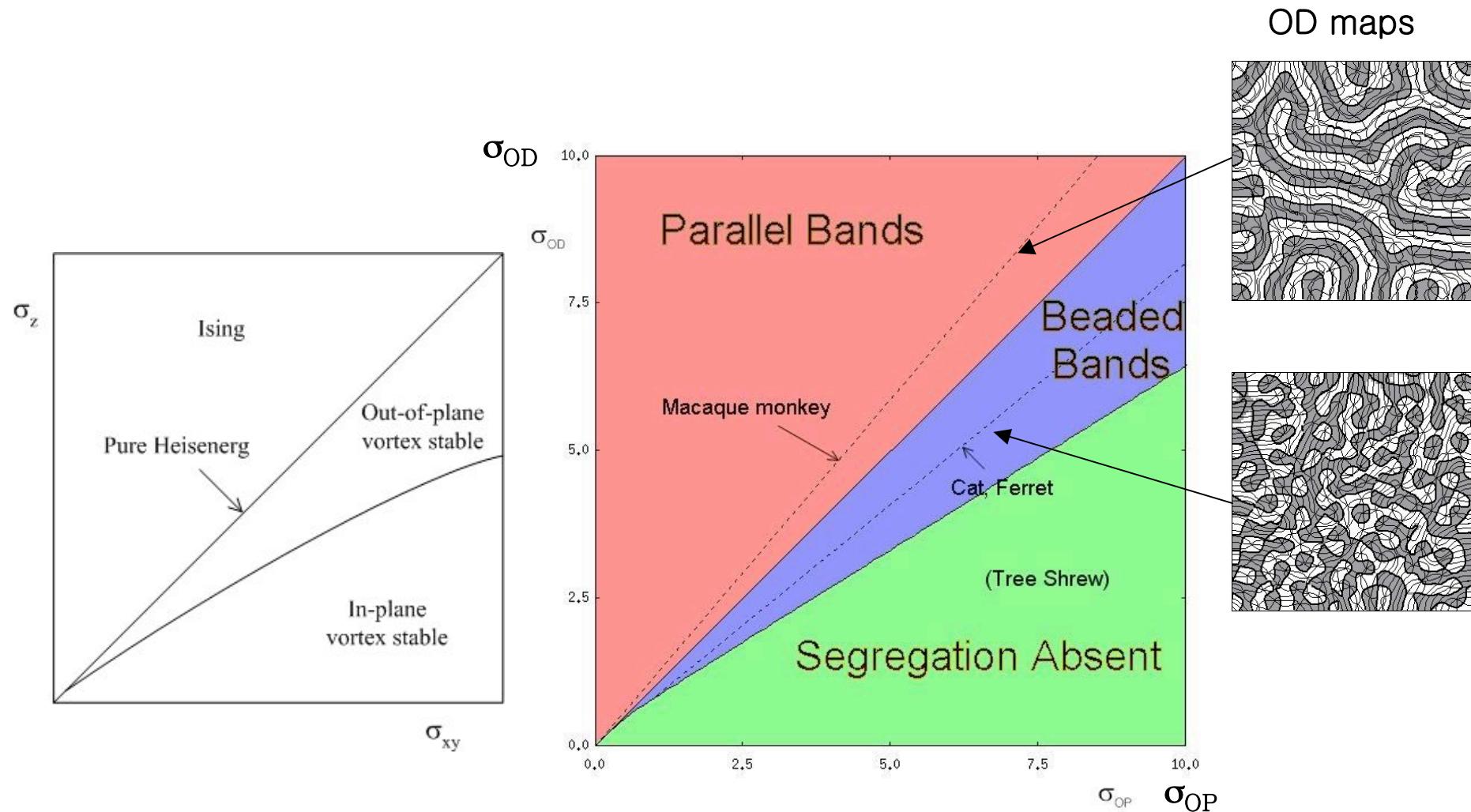


Wolf & Geisel, Nature 1998

$\lambda$  : anisotropy between OP and OD columns

$$\lambda \sim J_{OD} / J_{OP}$$

# Phase diagram of visual maps

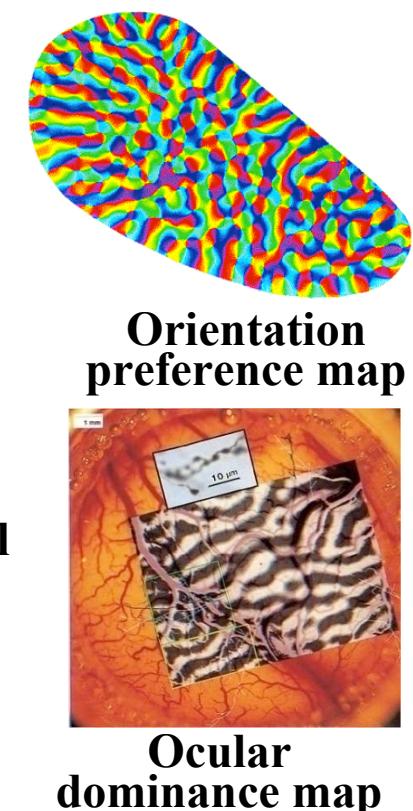
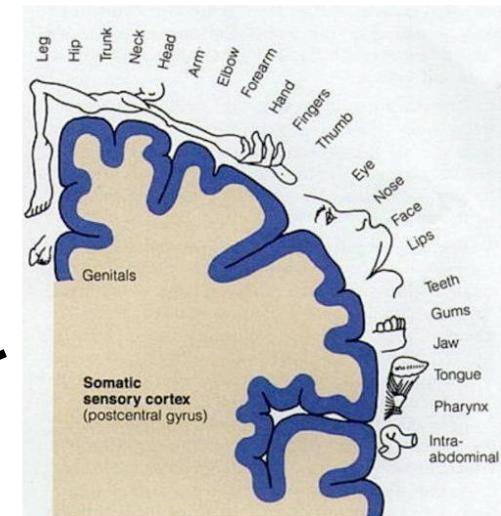
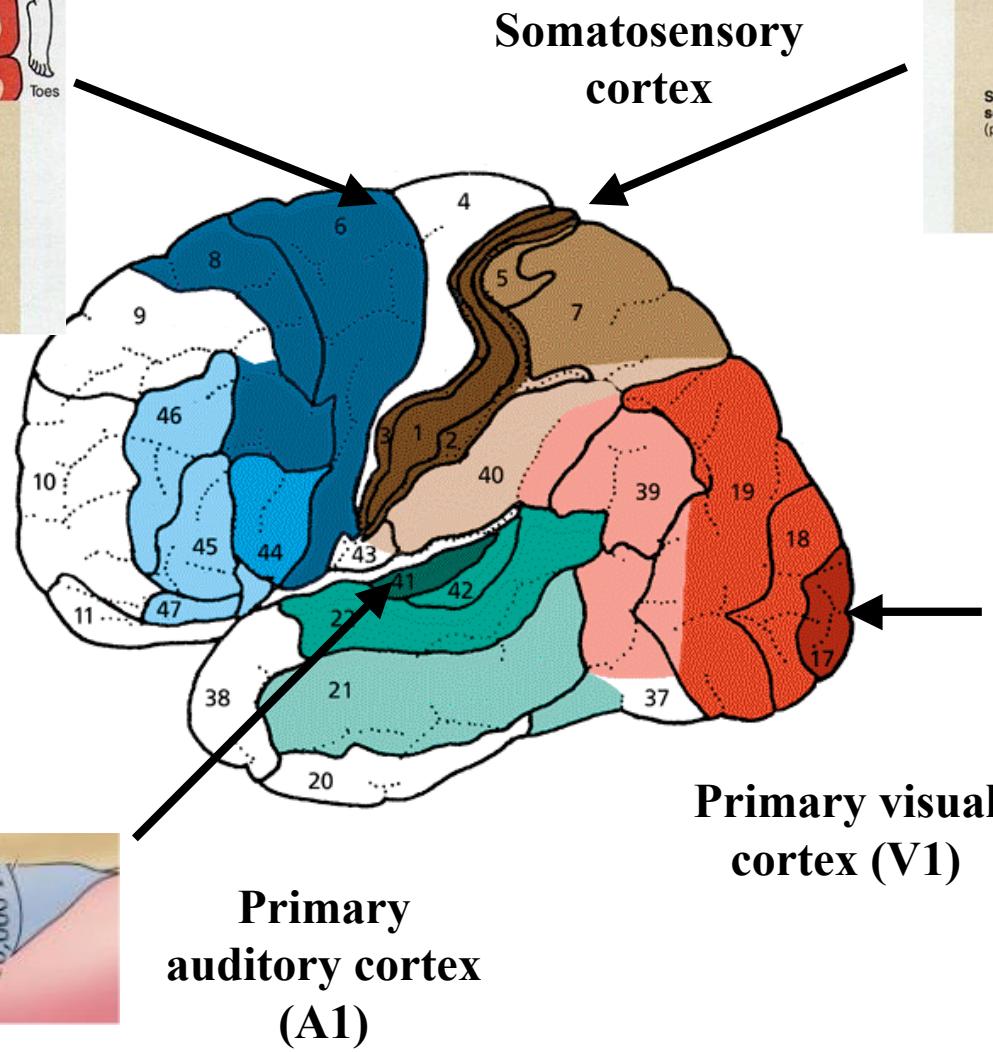
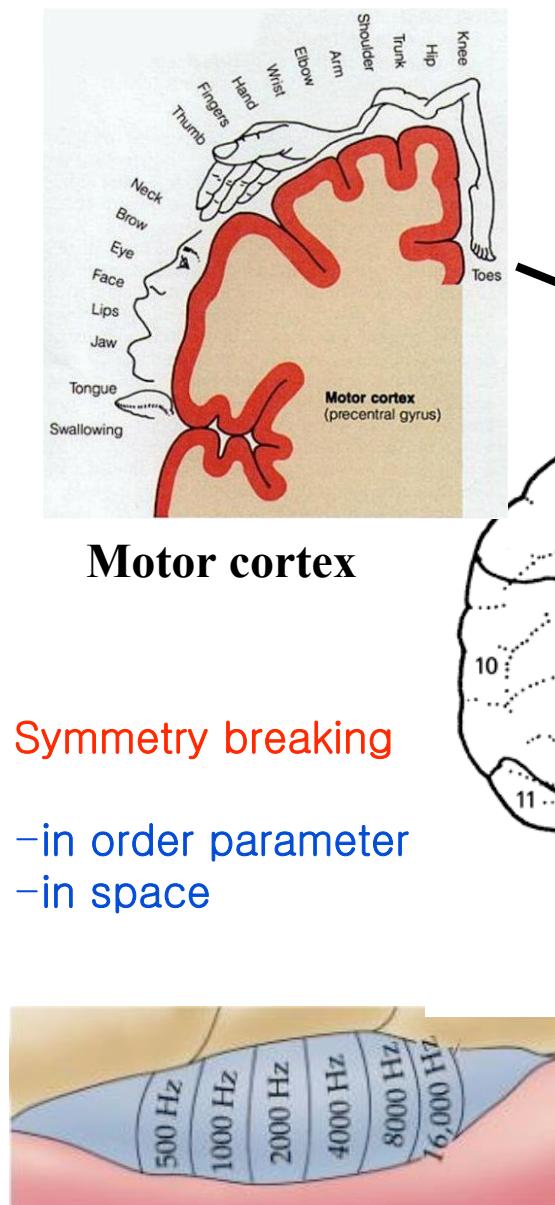


M.W. Cho & S. Kim, Phys.Rev.B , 2004

$\sigma_{OP}$ ,  $\sigma_{OD}$ : cooperation range  
in OP and OD columns

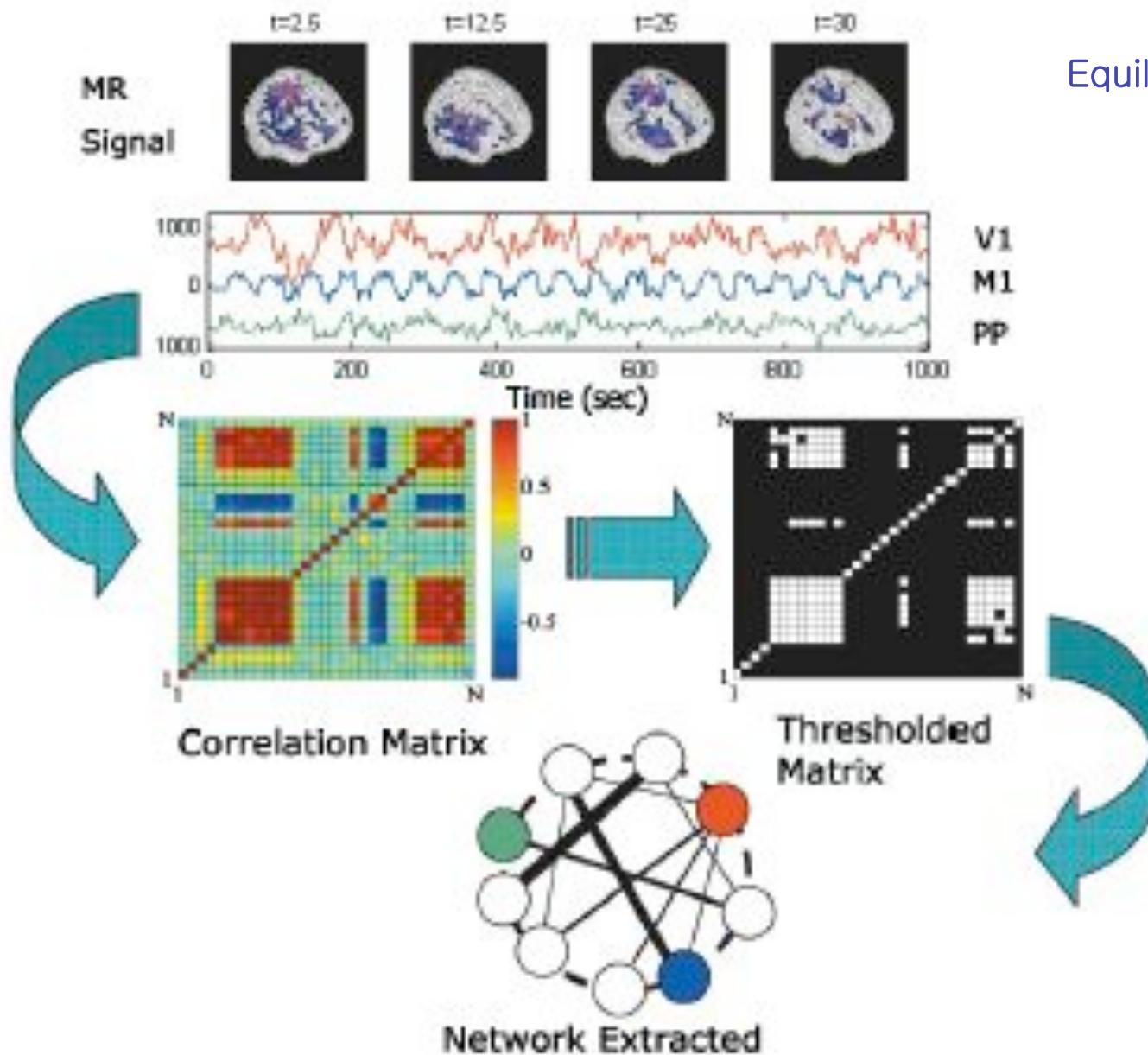
M.W. Cho & S. Kim, PRL, 2005

# Functional maps in a brain



# Functional network formation in brain

f-MRIs



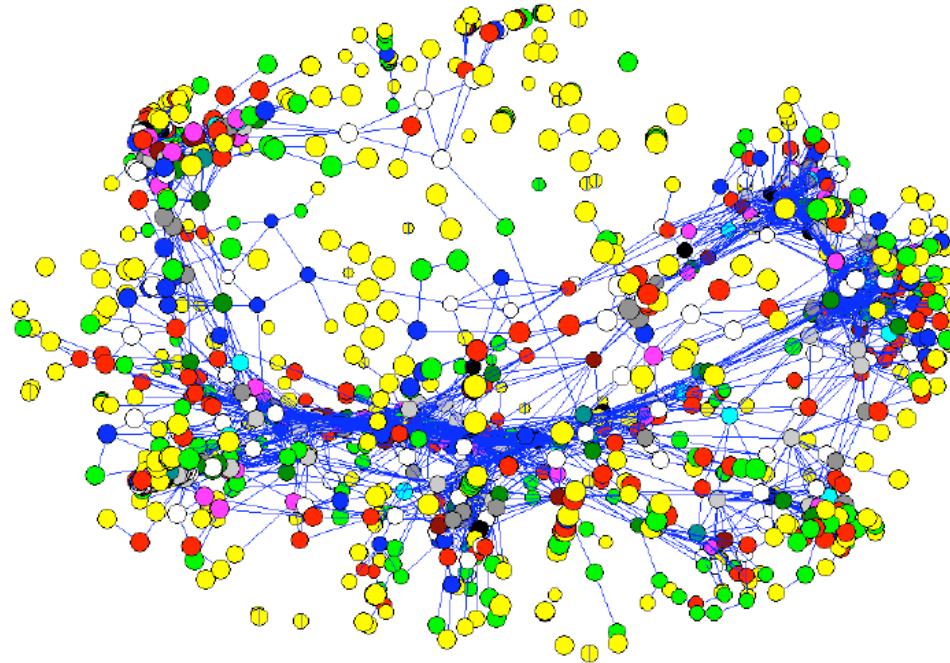
Equiluz et al, PRL64 2005

36x64x64 voxels  
400 x 2.5s

- Measure  $V(x,t)$
- Compute linear correlations
- Thresholded matrix
- Functional networks

# Functional networks in a brain

Equiluz et al, PRL64 2005  
f-MRI in humans



Robust for seven subjects at different tasks

Small-world

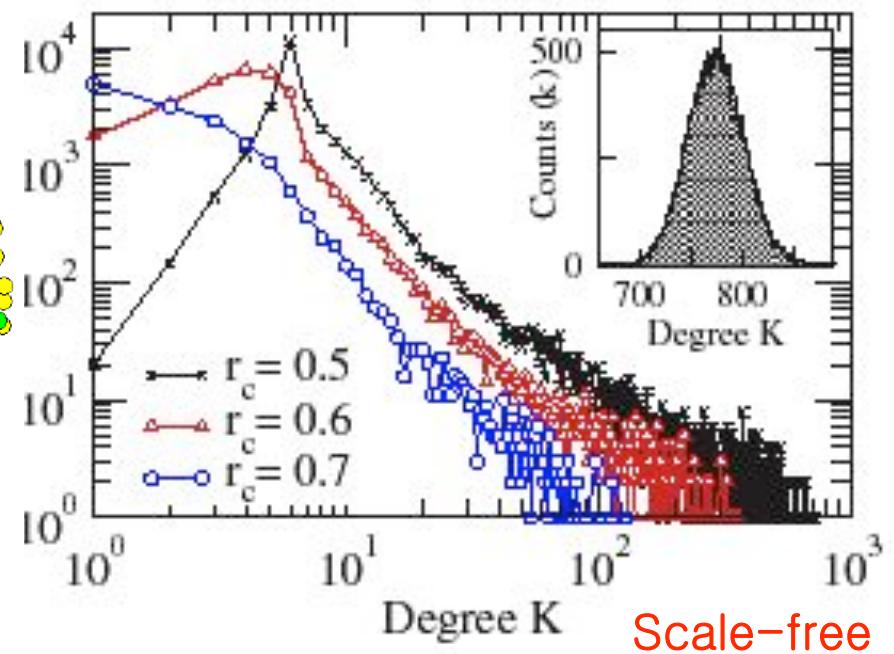


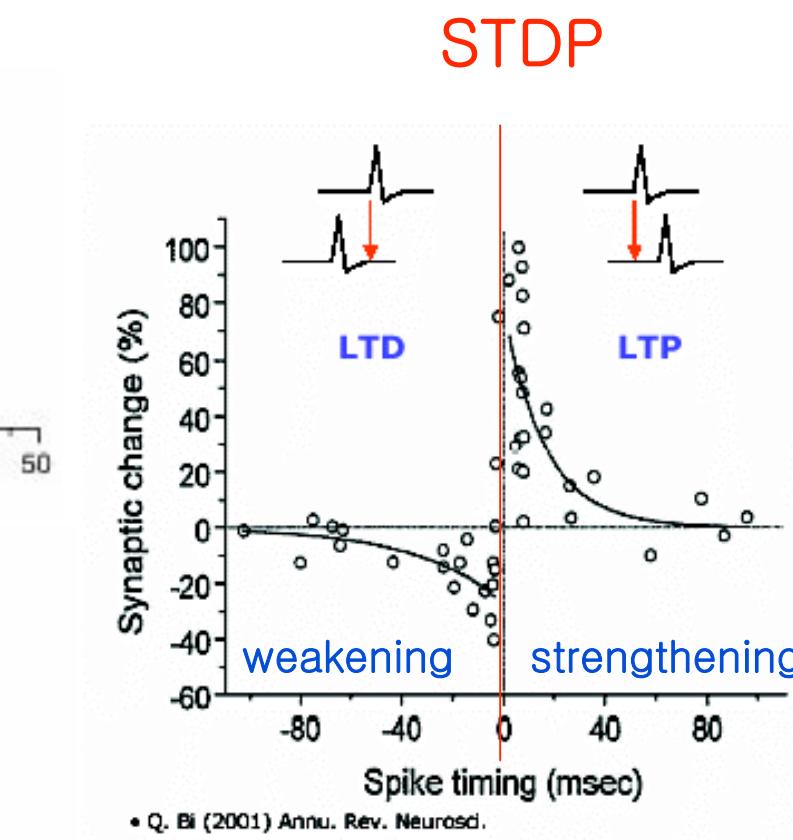
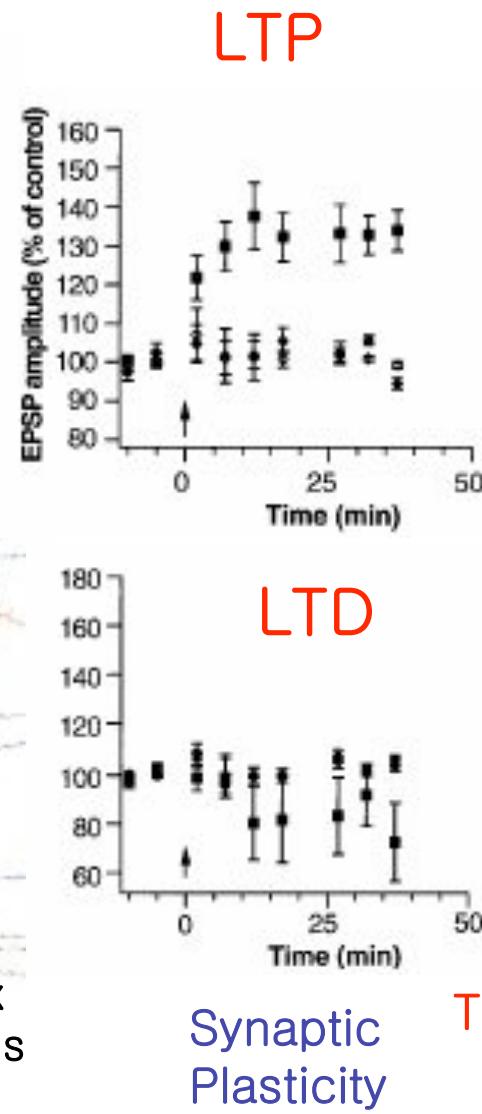
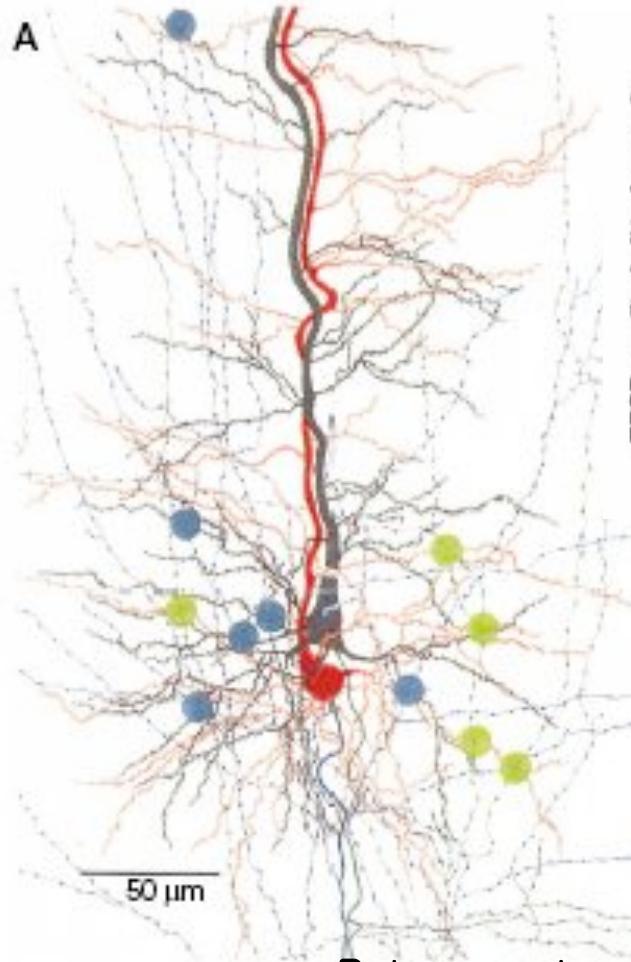
FIG. 2 (color online). Degree distribution for three values of the correlation threshold. The inset depicts the degree distribution for an equivalent randomly connected network.

Previous studies

$r_c$	$N$	$C$	$L$	$\langle k \rangle$	$\gamma$	$C_{\text{rand}}$	$L_{\text{rand}}$
0.6	31503	0.14	11.4	13.41	2.0	$4.3 \times 10^{-4}$	3.9
0.7	17174	0.13	12.9	6.29	2.1	$3.7 \times 10^{-4}$	5.3
0.8	4891	0.15	6.0	4.12	2.2	$8.9 \times 10^{-4}$	6.0

Network	$N$	$C$	$L$	$\langle k \rangle$	$\gamma$	$C_{\text{rand}}$	$L_{\text{rand}}$
C. Elegans	282	0.28	2.65	7.68	not applicable	0.025	2.1
Macaque VC	32	0.55	1.77	9.85	not applicable	0.318	1.5
Cat Cortex	65	0.54	1.87	17.48	not applicable	0.273	1.4

# Spike timing dependent plasticity (STDP)



Temporal order  $\leftrightarrow$  synaptic plasticity

# Model of a STDP neural network

Dynamic Model of Neural Networks  
+ Synaptic plasticity by STDP

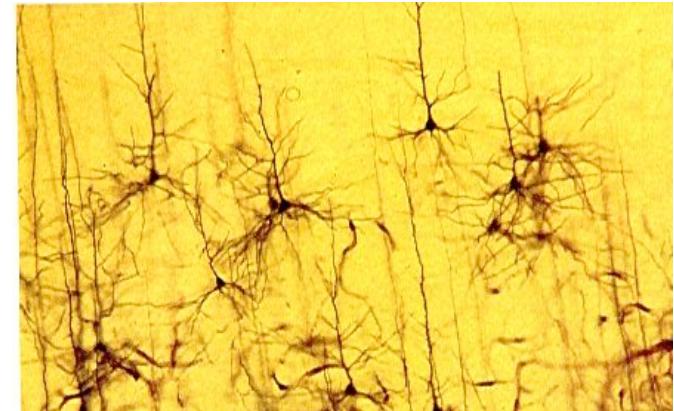
Neuron : FitzHugh–Nagumo model

$$\epsilon \dot{v} = I_{ion} + I_{syn} + I_{ext}$$

$$\dot{w} = v - w - b$$

$$I_{ion} = v(v - a)(1 - v) - w$$

$$\tau_{syn} \frac{dg_{ij}}{dt} = -g_{ij}$$



$$I_{syn}(t) = \sum_{j \neq i} [g_{ij}(t)(V - v_i(t)) + \bar{g}_{ij}(t)(\bar{V} - v_i(t))]$$

excitatory                      inhibitory

Initial all-to-all coupling

Synapse : STDP

$$W(\Delta t) = \begin{cases} A_+ \exp(-\Delta t/\tau_+) & \text{if } \Delta t > 0 \\ -A_- \exp(\Delta t/\tau_-) & \text{if } \Delta t < 0 \end{cases}$$

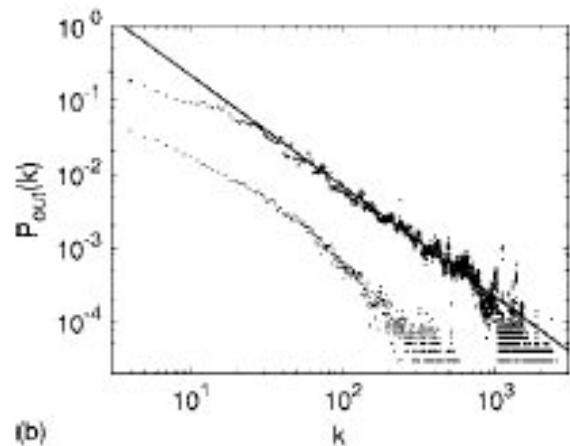
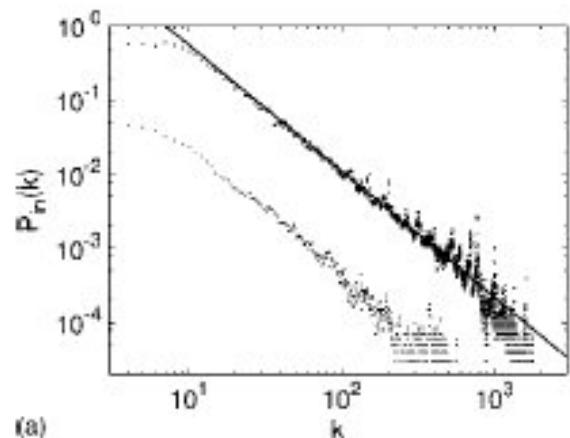
$$\Delta G_{ij} = G_{ij} \cdot W(\Delta t) \quad 0 < G_{ij} \leq G_{max}$$

$$\bar{G}_{ij}(t) = G_{inh}$$

Q: How does the functional network emerge through self-organization?

Dynamics  $\Leftrightarrow$  Structure  
– Activity dependent development

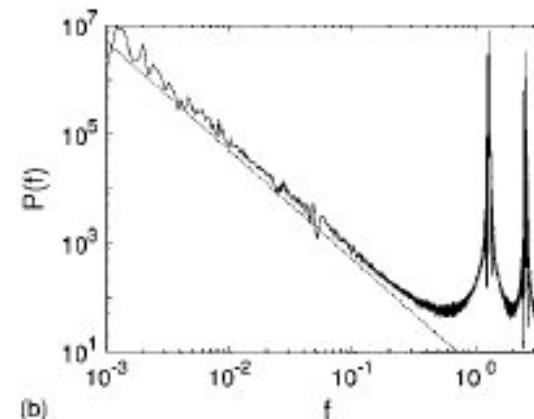
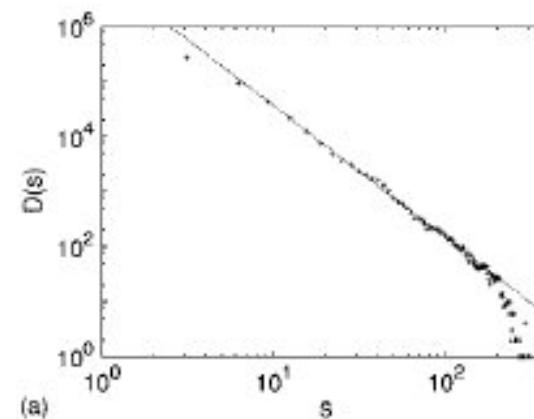
# Complex network formation through self-organization



- Sparse, small world
- Scale-free properties

Fast, high coherence & fast, strong synchronization

=> Dynamically more effective and structurally more robust

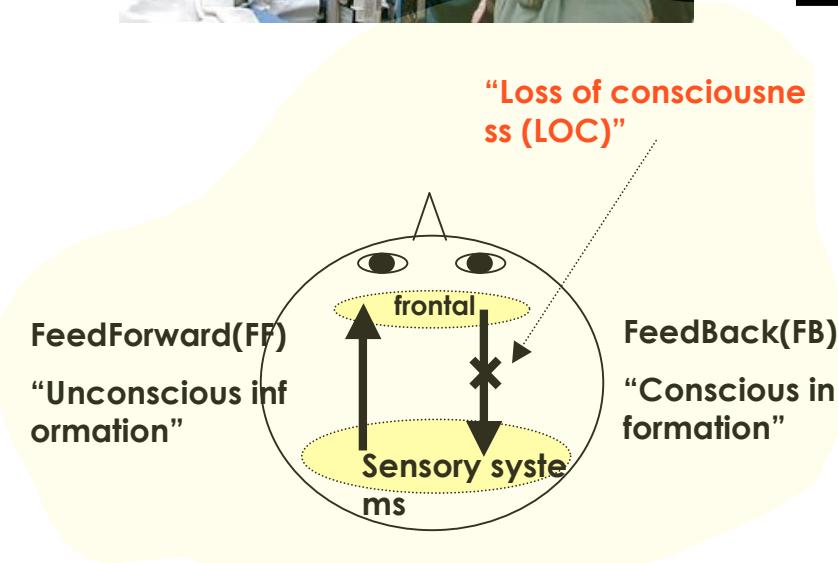


- Self-organized criticality

C. W. Shin & S. Kim, PRE, 2006

# Functional pathways in EEGs

Cooperating with Dep. Anesthesiology, Hyundai Asan Hospital, Seoul, Korea



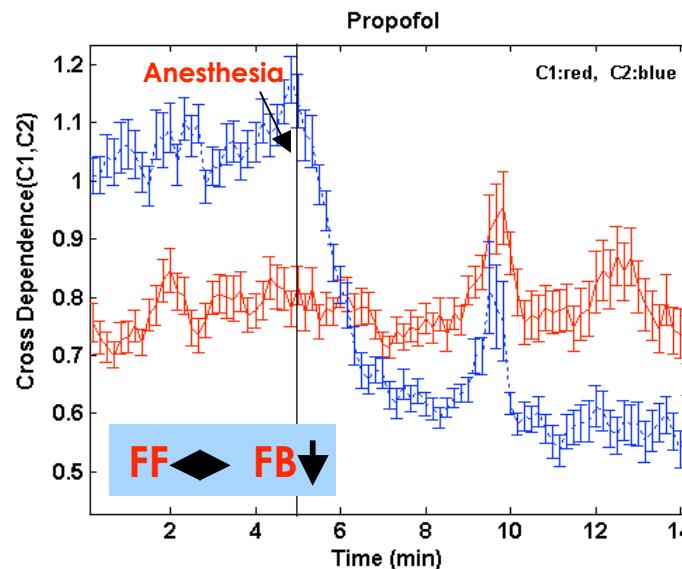
Want to understand the correlation between the feed back integration in the frontal-posterior network with conscious information processing in brain.



Anesthesia experiments with human subjects



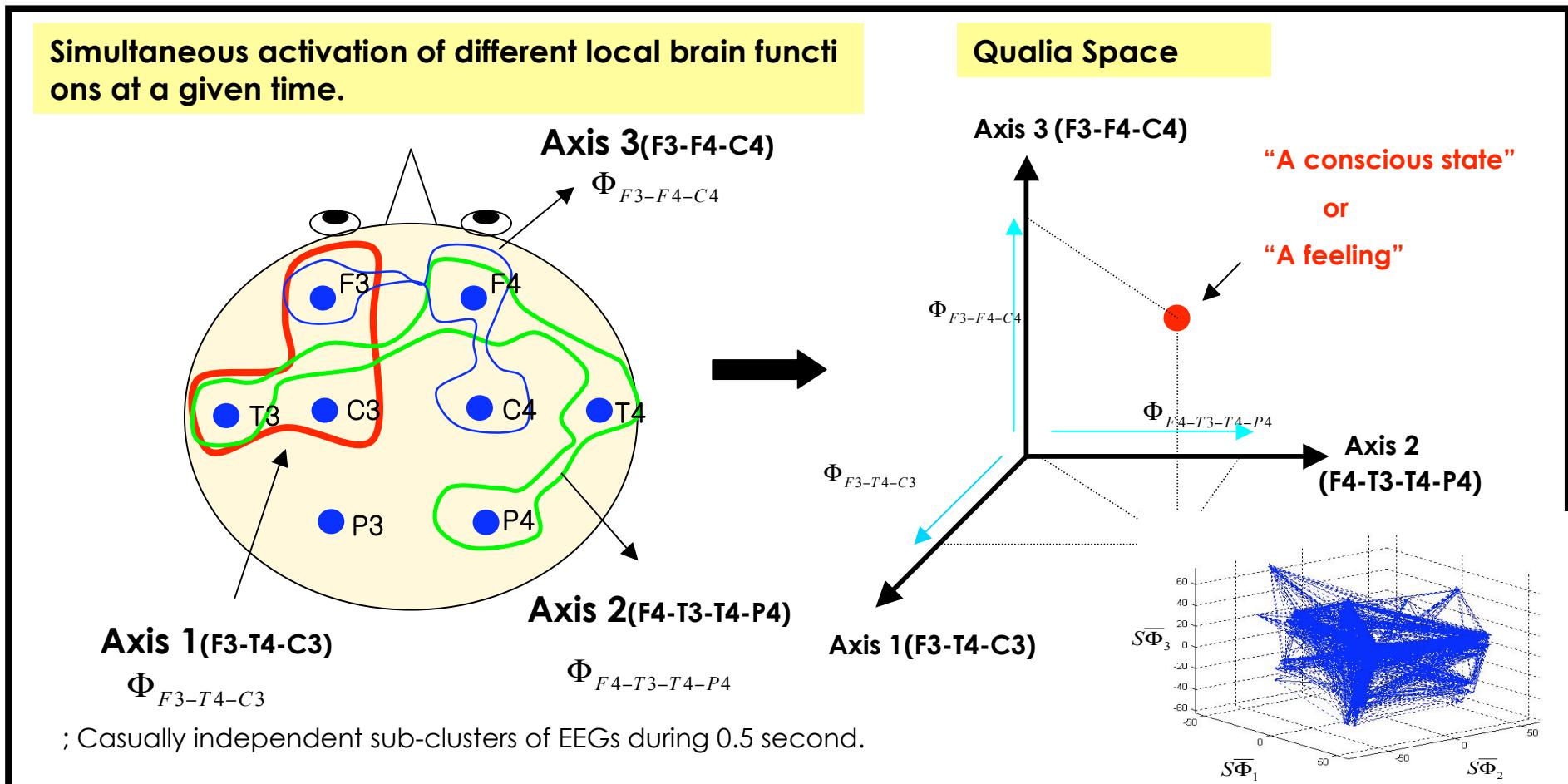
Analysis based on the Information Flow of EEG.



# Dynamics of consciousness in qualia space of EEG

## Research Goals:

1. Development of a way to reconstruct the qualia space with EEG
2. Investigation of time evolution of conscious state.



# Brain : Frontier of complex systems

- Studies of functional self-organization in brain (NCSL)
  - Map formation in visual cortex : M.W. Cho & S. Kim, PRL '04, PRL '05, PRB '05  
Functional development of asymmetry and area differentiation, preprint, '07
  - Functional complex network formation by STDP : C.W. Shin & S. Kim, PRE '06
  - Orientation tuning through synchronization : S.-G. Lee & S. Kim, PRE, '05
  - Functional networks in EEG : U.C. Lee & S. Kim, PRE, '06, preprint, '07
- Brain : Paradigm for complex biological systems
  - Highly nonlinear neuron, and complex network structure
  - Self-organization and activity dependent neural plasticity
- Fertile interdisciplinary grounds for new quantitative methods
  - Nonlinear Dynamics + Statistical Mechanics + Computations
  - ⇒ Understanding biocomplexity of the nervous system & its role in brain function

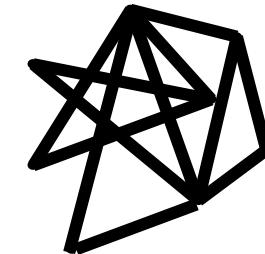
Work in progress and challenges ahead!

\* One of the 10 unsolved problems in physics!

\*IOP,1999

# Complex networks

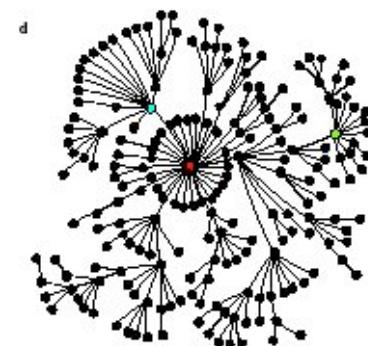
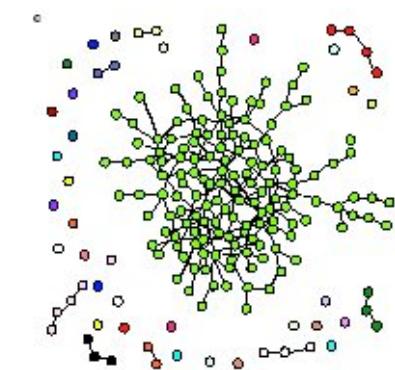
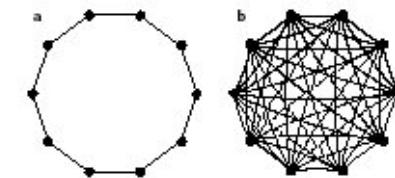
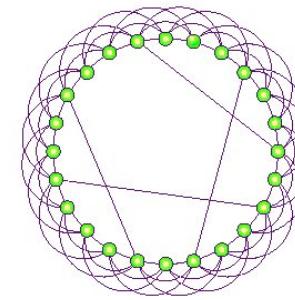
- Networks
  - node → network
  - regular, random, complex networks
  - Bionetworks, www, internet, airport, social relations



## Q. Network structure vs functions?

Eg. Robustness, efficiency

- Complex networks
  - Small World networks
    - Short path length : “six deg. of separation”
    - High clustering : A->B, B->C => A->C
  - Scale-Free networks
    - Degree distribution : Power-law  $p(k) \sim k^{-\gamma}$
    - <cf> random networks : Poisson distribution



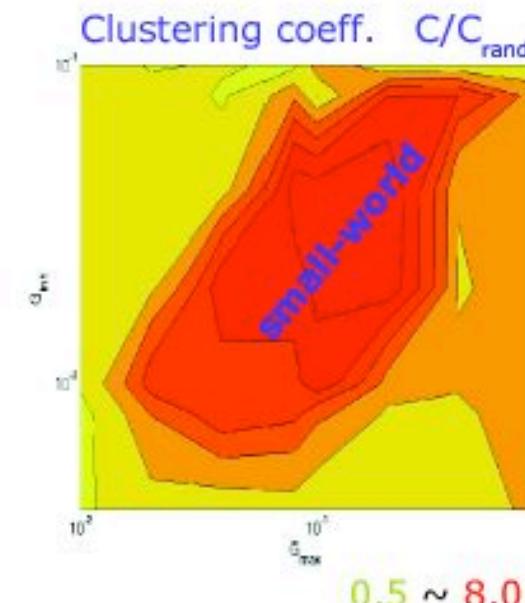
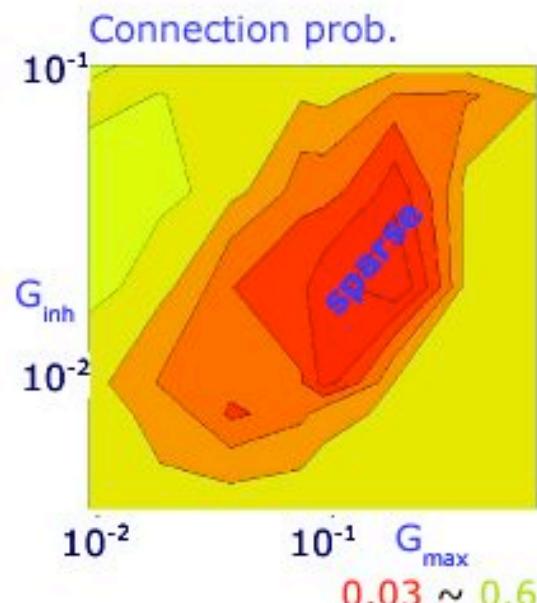
Strogatz, Nature 410, 268 (2001)

# Functional Structure by STDP

Globally coupled network

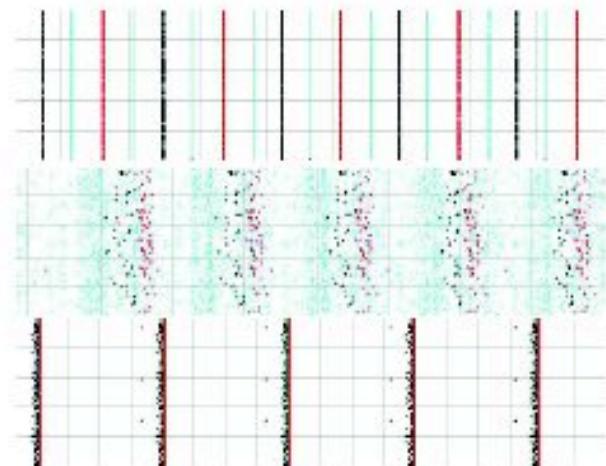
STDP

Functional Structure

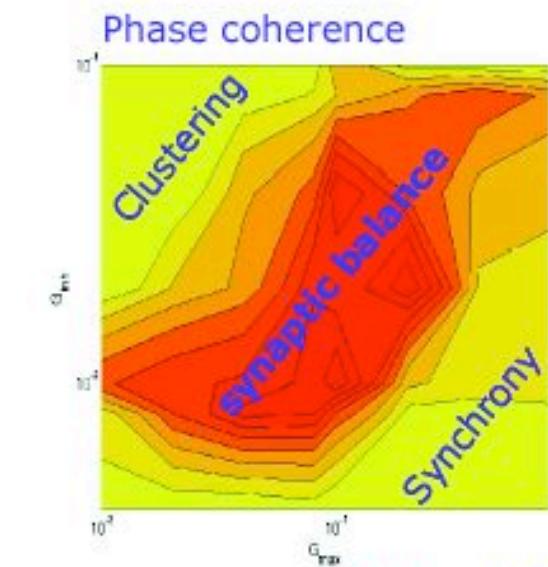


Fraction of connections between  
maximal connected neighbours

cluster



synchrony

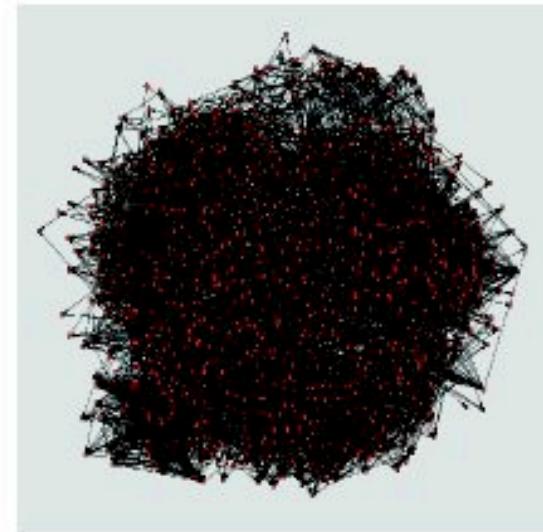


Degree of phase coherence  
among clusters

# Small-world and Scale-free Network

## Small-world properties

- clustering coefficient
  - $C = 0.23 \gg C_{\text{rand}} = 0.03$
- Average path length
  - $L = 3.19 \sim L_{\text{rand}} = 2.03$

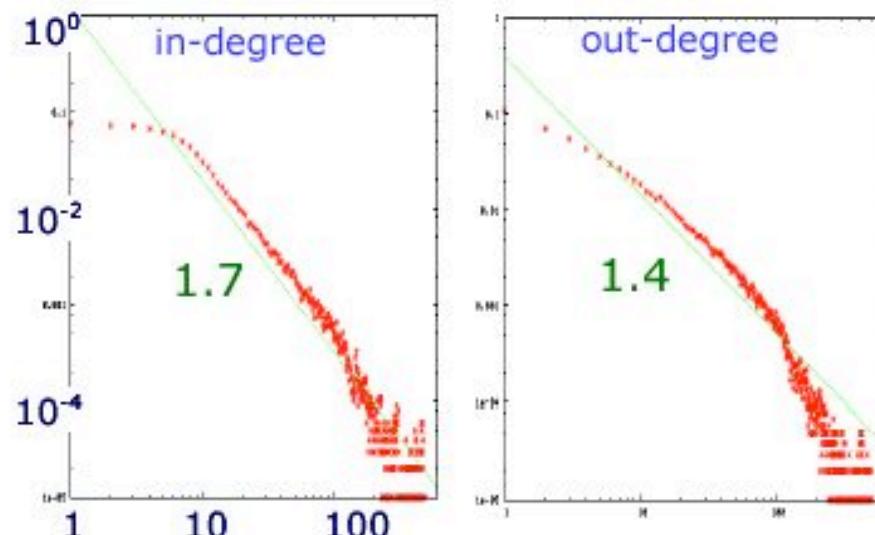


## Scale-free properties

- Degree distributions

$$P_{in}(k) \sim k^{-\gamma_{in}}$$

$$P_{out}(k) \sim k^{-\gamma_{out}}$$

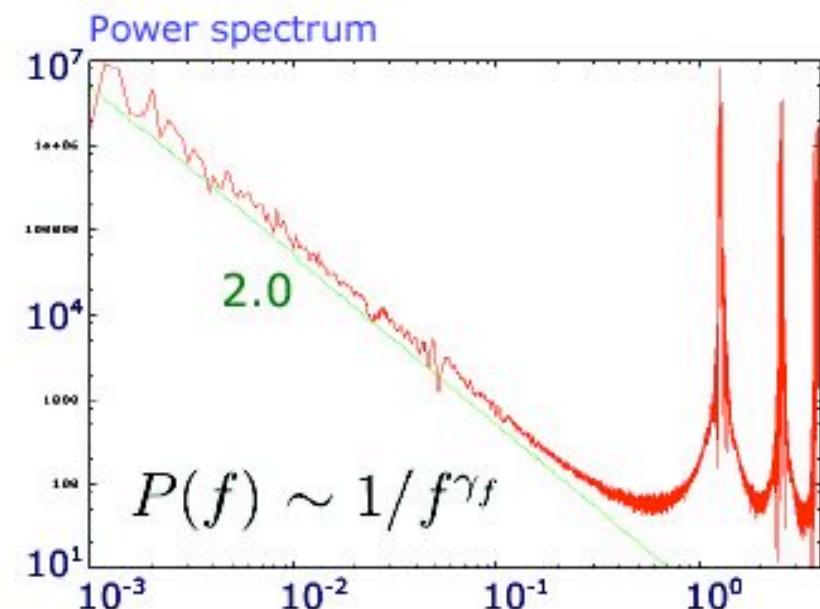
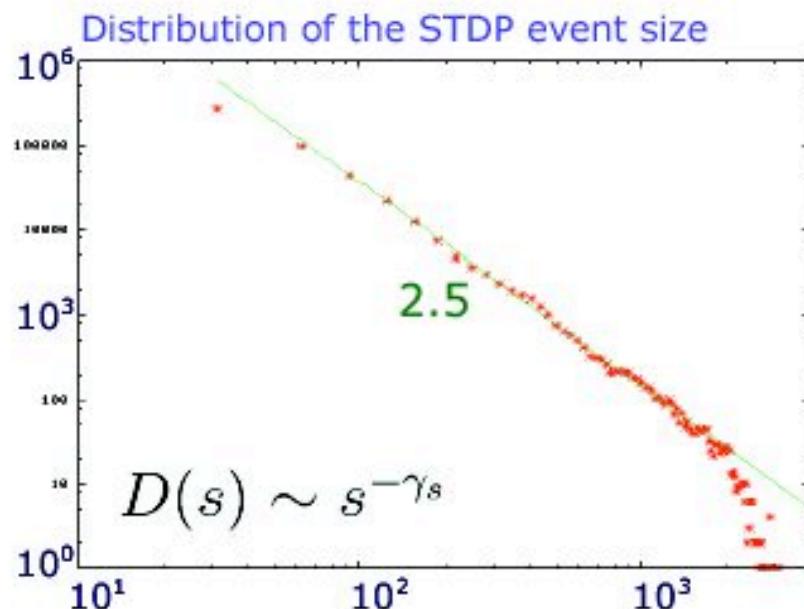


$$G_{\max} = 0.2, G_{inh} = 0.02$$

# Self-organized Criticality

Quasi-steady state in network properties

## Fluctuation of synaptic coupling strength



- Functional structure lies in a self-organized critical state
- STDP events have no long-time correlation

Phase flips propagate in an avalanche manner

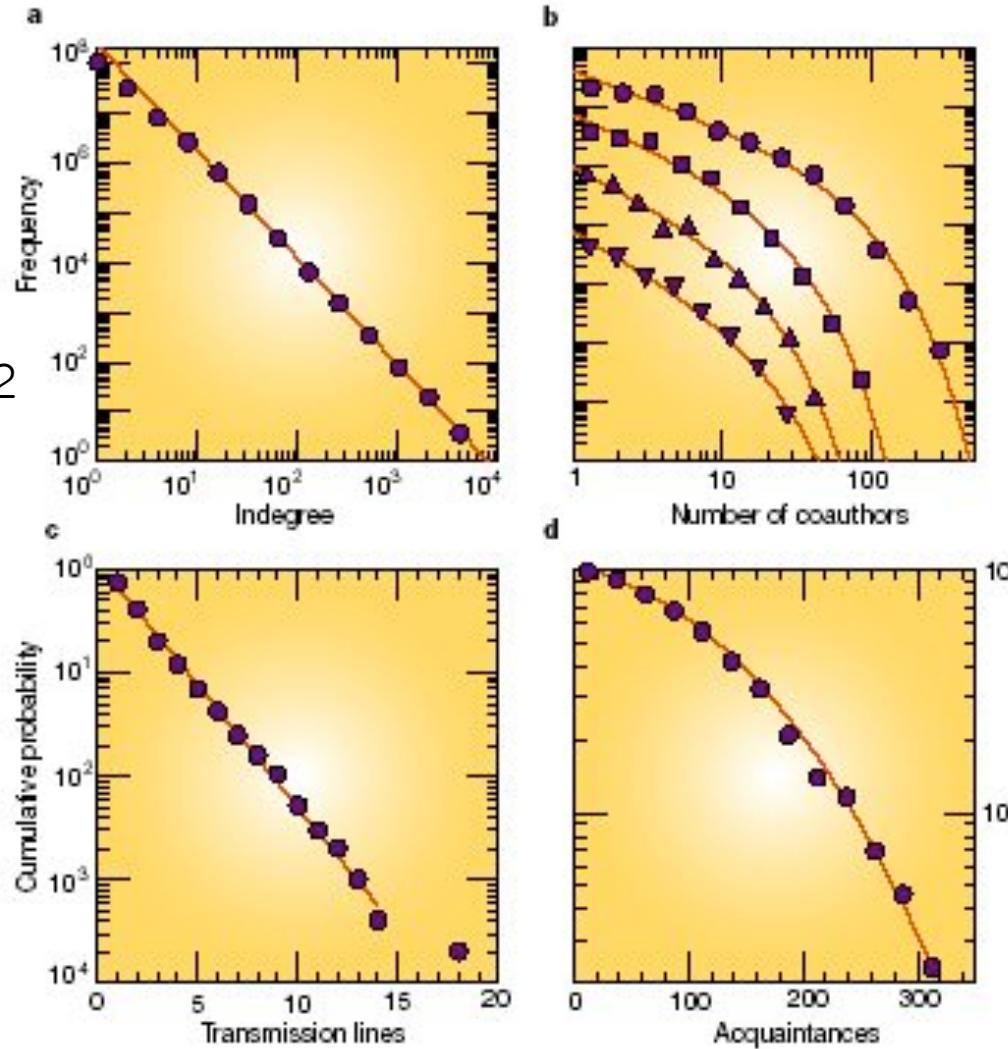
$\langle cf \rangle$  sandpile

# Diverse networks

WWW

Scale-free  $\gamma=2.2$

Power grid  
exponential

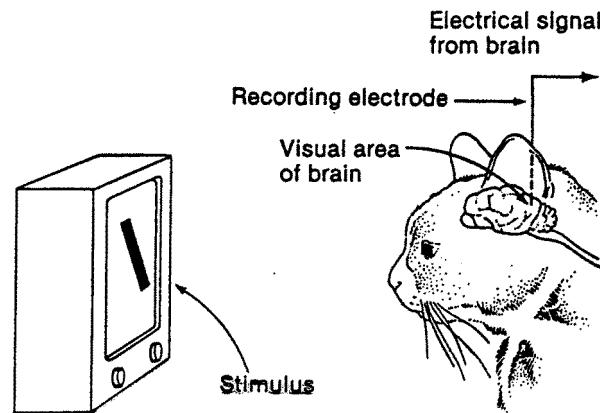


Coauthorship

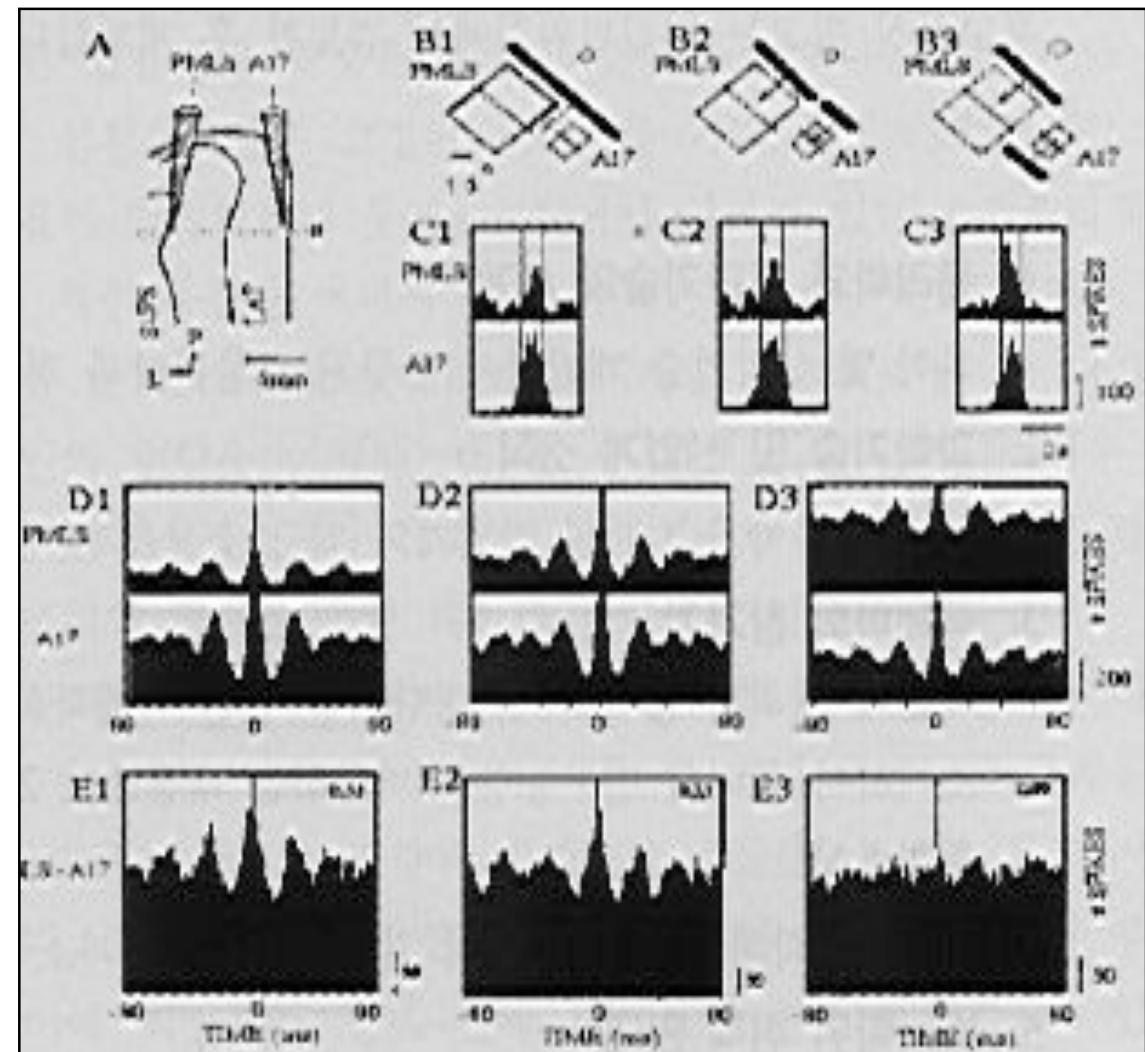
(arxiv.org., SPIRE  
S, NCSTRL)  
Exponential cutoff  
f

Social (Mormon)  
Gaussian

# Synchrony in cat visual cortex



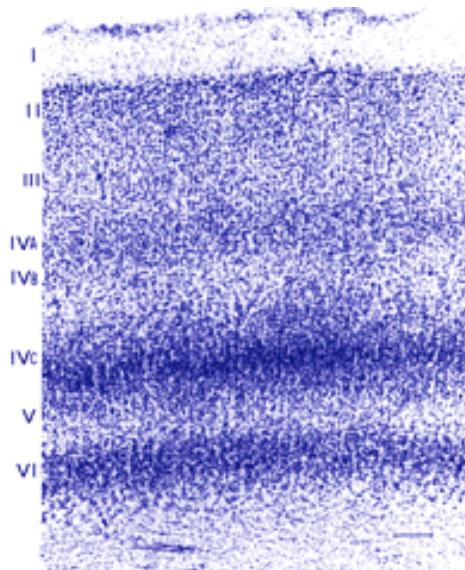
- Neurophysiological experiments on neuron pairs far apart
- Observation: Stimulus dependent, long-range synchronization
- Synchronization – same orientation columns
- Non-synchronization – different orientation columns



Engel et al, 1990

# Orientation columns in visual cortex

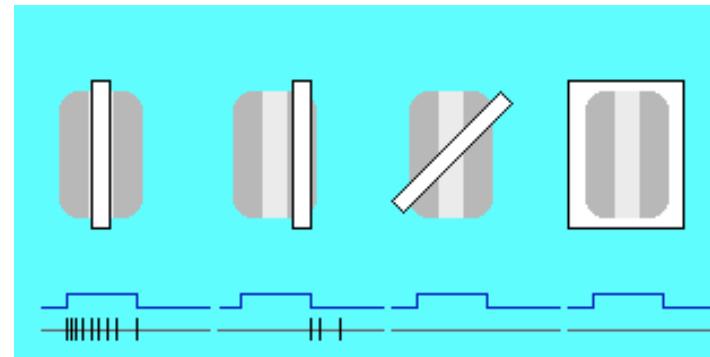
Q: How do neurons become selective to a stimulus?



Visual cortex

Orientation selectiviey

Hubel & Wiesel, 1959



Elongated patterns  
of convergence fro  
m thalamic inputs

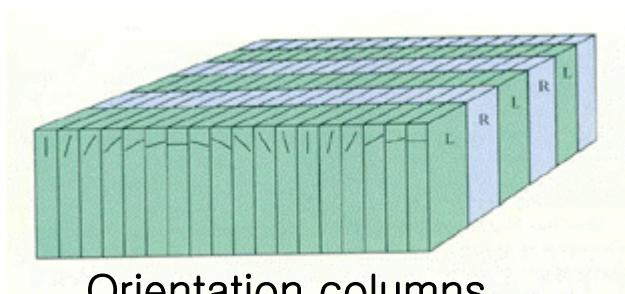
Observations:

LGN input weakly tuned. Fester et al, 1990

Orientation selectivity is sharp  $\sim 18^\circ$

Q: How is the weakly tuned input from LGN sharpened?

Orientation tuning  $\leq$  Intracortical connections?  
relevance to synchronization?

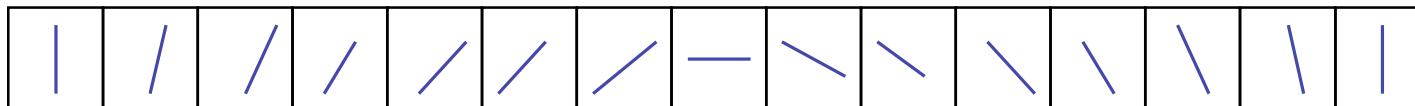


Orientation columns

Q: Why is the orientation tuning not dependent on contrast?

Contrast invariance

# Orientation Column Model



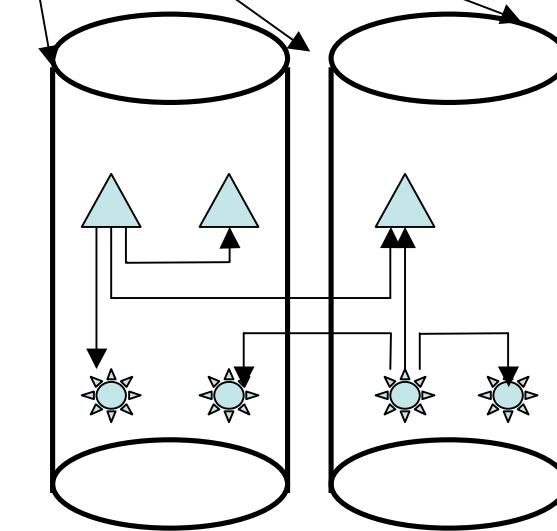
Spiking neuron model at subcolumnar level

## Synaptic connection model

- All-to-all excitatory within a column
- All-to-all excitatory between neighboring columns:  
Connection strength 15 % smaller
- Inhibitory cells in the same column receive same inputs
- Global inhibitory to all excitatory cells
- Global inhibitory to all inhibitory cells

## Neuron & synapse models

- Hodgkin-Huxley neurons:
- $\alpha$  synaptic coupling (Jack, 1975)
- Correlated noisy current :  
Ornstein-Uhlenbeck



Orientation columns

Lee, Tanaka, Kim, 2004

- A column → 12 excitatory cells + 3 inhibitory cells
- Angular difference : 12.85

225 neurons

# Orientation tuning

S.G. Lee & S. Kim, 2004.

