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Modern Physics
Spring 2005

Assignment 2

1. Consider a particle of mass m which can move freely along the x-axis anywhere from $x = -\frac{a}{2}$ to $x = +\frac{a}{2}$, but which is strictly prohibited from being found outside this region. This corresponds to what is called an infinite square well potential $V(x)$ given by $V(x) = 0$ for $-\frac{a}{2} < x < +\frac{a}{2}$ and $V(x) = \infty$ elsewhere. If we solve the Schrodinger equation for this $V(x)$, one of the solutions is given by

$$\Psi(x,t) = \begin{cases} A\cos\left(\frac{\pi x}{a}\right)e^{-iEt/\hbar} & -\frac{a}{2} < x < \frac{a}{2} \\ 0 & |x| \geq \frac{a}{2} \end{cases} \qquad (a > 0)$$

   (a) Find A so that the function $\Psi(x,t)$ is properly normalized.

   (b) In the region where the potential $V(x) = 0$, the Schrödinger equation reduces to
   $$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}.$$
   Plug $\Psi(x,t) = A\cos\left(\frac{\pi x}{a}\right)e^{-iEt/\hbar}$ into this relation to show $E = \frac{\pi^2\hbar^2}{2ma^2}$.

   (c) Show that $\langle P\rangle = 0$.

   (d) Evaluate $\langle x^2\rangle$ for this wavefunction. You will probably need an integral table for this. (Of course, you can always try contour integration or some such. But, that is beyond this course.)

   (e) Evaluate $\langle P^2\rangle$ for the same wavefunction. You will not need an integral table if you use the fact that the function is normalized. Of course, a brute force computation will yield the same result as well.

2. If you are a careful student who pays attention to minute details, you may have realized that A can only be determined up to the argument, or up to the sign if you assume A is a real number.

   (a) What is the implication of this fact as to the uniqueness of wavefunction?

   (b) What is the implication of this fact as to the probability density $\Psi^*(x,t)\Psi(x,t)$? How about $\langle P\rangle$, $\langle P^2\rangle$, and $\langle x^2\rangle$?