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Modern Physics
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Name: ______________________________

# Final Examination

1. Consider a potential $V(x, t)$.

   (a) Write down, but do not derive, the time-dependent Schrödinger equation for $V(x, t)$. This is a one-dimensional case, and the only variables are the spatial variable $x$ and the temporal variable $t$. [1 pt.]

   (b) Write down, but do not derive, the time-independent Schrödinger equation for $V(x)$, that is, $V$ is now a function of $x$ only. This is a one-dimensional case, and the only spatial variable is $x$. [1 pt.]

   (c) If two functions $K(x)$ and $L(y)$ satisfy $K(x) = L(y)$ for all values of $(x, y)$ and $x$ and $y$ can be any real number, prove that both $K(x)$ and $L(y)$ are constant. [2 pts.]

2. The potential for the infinite square well is given by

$$V(x) = \begin{cases} \infty & |x| > \frac{a}{2} \\ 0 & |x| < \frac{a}{2} \end{cases} ,$$

   and the solutions for the time-independent Schrödinger equation given in the class were

$$\begin{cases} \psi_n(x) = B_n \cos k_n x & \text{where} \quad k_n = \frac{n\pi}{a} \quad n = 1, 3, 5, \ldots \\ \psi_n(x) = A_n \sin k_n x & \text{where} \quad k_n = \frac{n\pi}{a} \quad n = 2, 4, 6, \ldots \end{cases} .$$

(a) Find a normalized solution for $n = 2$ for which the normalization constant $A_2$ is purely imaginary. [2 pts.]

(b) What is the expectation value of the position $x$ for this solution? Use the full wavefunction to solve this problem. [2 pts.]

(c) Plug the above solution into the lefthand side of the time-independent Schrödinger equation to determine $E_2$, the total energy associated with this state. [2 pts.]

3. Answer the following questions about the hydrogen atom.

(a) Consider $n = 3$.

i. What $l$ values are possible? [2 pts.]

ii. For each value of $l$, what $m_l$ values are possible? [2 pts.]

iii. How many degenerate states do we have for $n = 3$? [2 pts.]

(b) What is the energy of a photon emitted when the electron drops from the 3rd highest energy level ($n = 3$) to the ground state ($n = 1$)? Leave $\mu$, $e$, $\pi$, $\varepsilon_0$, and $\hbar$ as they are. [2 pts.]

4. Try a series solution

$$f(x) = \sum_{j=0}^{\infty} a_j x^j$$

for the differential equation

$$\frac{df(x)}{dx} + f(x) = 3x^2 + 8x + 3,$$

and show that

$$a_j + (j+1)a_{j+1} = 0 \quad \text{for} \quad j \geq 3.$$

[2 pts.]