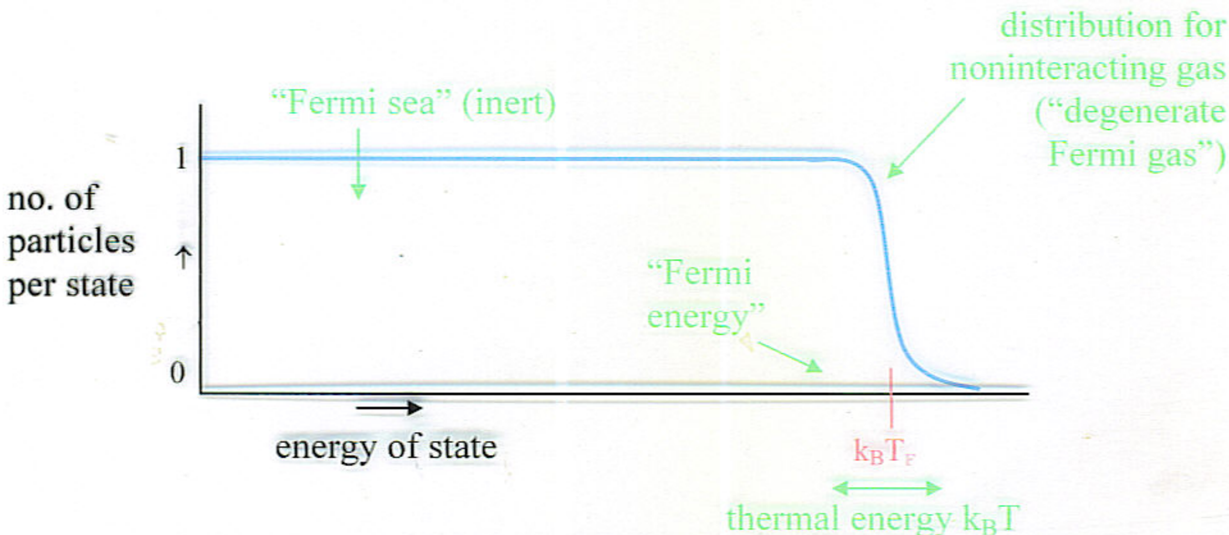
SUPERFLUID ^{3}HE: SOME PRE-HISTORY

^{4}He: below 2K, superfluid

ELECTRONS IN METALS	LIQUID ^{3}HE
since ancient times	since ~ 1950
charged	neutral

Particles of spin ½ ⇒ Fermi statistics



Landau (1957): interactions don't change picture qualitatively (in "normal" phase) ("degenerate Fermi liquid")

$T_F \sim 10^4 - 10^5$ K	$T_F \sim 5$ K
at T $\lesssim$ 20 K.	at T $\lesssim 10^{-3}$ K.
superconductivity	superfluidity??
(in some metals)	

MORE PREHISTORY

Theory of superconductivity:

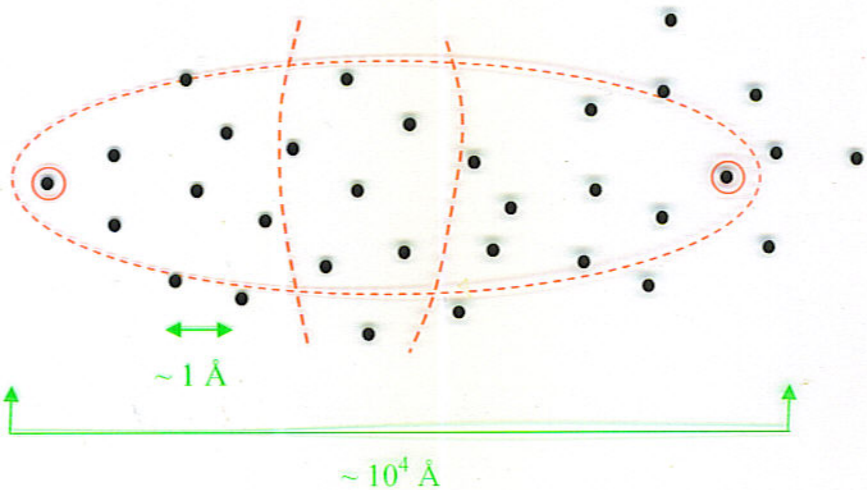
(a) phenomenological (V. L. Ginzburg, A. A. Abrikosov, et al., 1950-1955):

macroscopic wave function

(b) microscopic (Bardeen et al., 1957):

electrons in energy shell of width ~ $k_B T_C$ around Fermi energy form Cooper pairs

critical temp., $\lesssim$ 20K



Crucial feature of BCS theory: ALL COOPER PAIRS MUST BEHAVE IN EXACTLY THE SAME WAY!

(GL "macroscopic wave function" is just the common center-of-mass wave function of all the pairs)

In BCS theory, "internal" wave function of pairs trivial: (" ^1S_o")

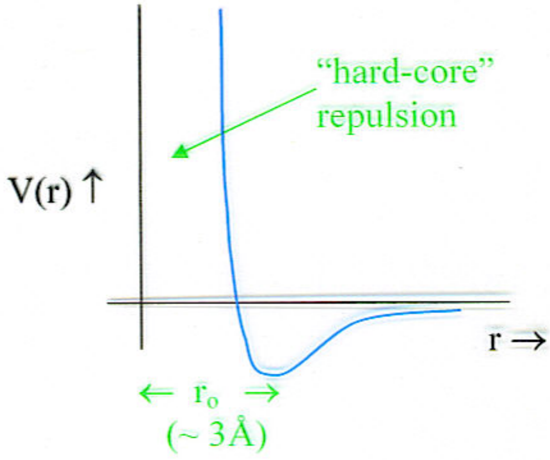
$$\psi\ (\underset{\sim}{r}_1 \underset{\sim}{r}_2 : \sigma_1\sigma_2) \sim \frac{1}{\sqrt{2}}\ (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2)\ f\,(|\underline{r}_1 - \underline{r}_2\,|)$$

spin singlet

spherically symmetric ($\ell = 0$)

NO INTERNAL ("ORIENTATIONAL") DEGREES OF FREEDOM

EARLY THEORETICAL WORK ON POSSIBLE COOPER PAIRING IN LIQUID ^{3}HE



$r \sim r_0$, $p \sim p_F$ ($\equiv \sqrt{2mk_B T_F}$)
$\Rightarrow$ relative angular momentum
$\ell \equiv (p_F r_0/\hbar) \neq 0$
(prob. 1 or 2)

Pauli principle: $\begin{cases} \ell = 0,2,4\ldots & S = 0 \quad \text{(singlet)} \\ \ell = 1,3,5\ldots & S = 1 \quad \text{(triplet)} \end{cases}$

in general, $\ell \neq 0 \Rightarrow$ relative (internal) wave function of pair has orientational degree(s) of freedom!

"equal spin pairing"

Anderson & Morel (1961): explore in detail case $\ell = 2$, and a special case of $\ell = 1$: only ↑↑ and ↓↓ pairs form, and have the same orbital ang. momentum in direction $\hat{\underset{\sim}{\ell}}$ ("ABM" state) Physical properties anisotropic.

Vdovin, Balian & Werthamer (1963): in $\ell = 1$ case all spin components ($\uparrow\uparrow, \downarrow\downarrow, \frac{1}{\sqrt{2}}\uparrow\downarrow + \downarrow\uparrow$)) can form: in fact for any given pair, $\underset{\sim}{L} = -\underset{\sim}{S} \Rightarrow J = 0$. ("BW" state). All physical properties isotropic. More stable than any ESP state.

"^3P_o"

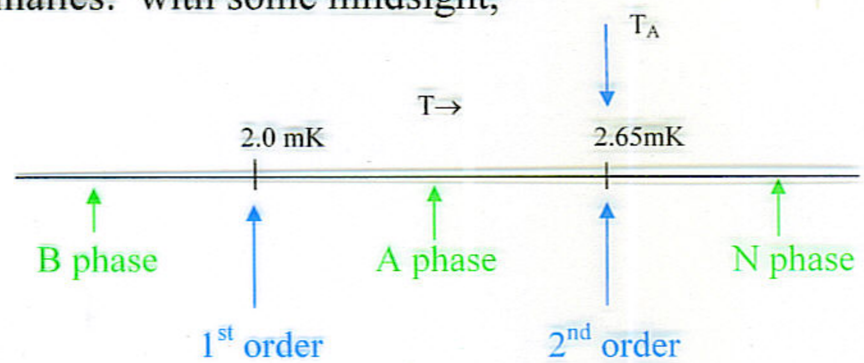
Theoretical expectation c. 1964:

Liquid ^{3}He may form Cooper pairs, either with ℓ = even (spin singlet) or with ℓ = odd (BW state). In either case, χ reduced and all magnetic properties isotropic. T_c difficult to predict.

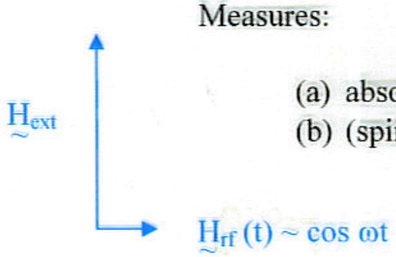
THE EXPERIMENTS OF 1971-72 (D. D. Osheroff, R. C. Richardson, D. M. Lee…(Nobel prize 1996)):

Mixture of liquid and solid ^{3}He, T < 3 mK.
(so only temperature varied).

First expts: pressurization (P as f(t))
2 anomalies: with some hindsight,



NMR:



Measures:

(a) absorption as f(ω)
(b) (spin) susceptibility χ

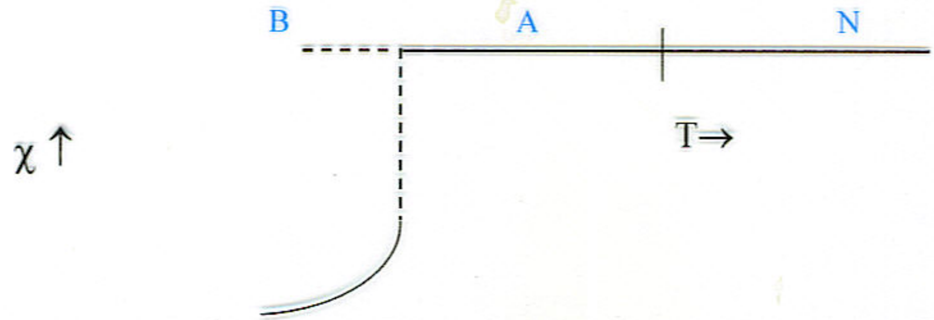
In N state:

χ independent of temperature, value as expected for degenerate Fermi liquid

Abs^n shows v. sharp peak at free-atom Larmor frequency: $\omega_{res} = \gamma H_{ext}$

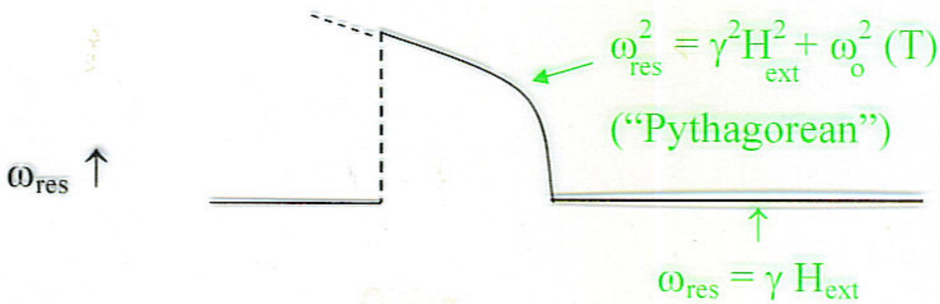
gyromagnetic ratio of free ^{3}He atom, ~ 3000 Hz/G

NMR in the new phases:

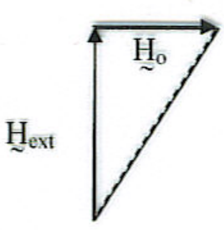


Not necessarily mysterious: e.g. A phase could be an ESP state (only ↑↑,↓↓ pairs ⇒ no reduction in χ), B could be singlet or BW (some ↑↓ pairs, so χ reduced) [but: why is ESP ever stable?] -

But: what about the resonance frequency?



$$\omega_0^2(T) \approx A(1 - T/T_A), \qquad \frac{A}{(2\pi)^2} \cong 5 \times 10^{10}\ \text{Hz}^2$$



$(\equiv \omega_o(T)/\gamma)$

Need $H_o \sim 30$G. But, only spin-nonconserving force in problem is nuclear dipole-dipole interaction, and max. associated field is < 1G!

IS THIS THE FIRST INDICATION OF A RADICAL BREAKDOWN OF QUANTUM MECHANICS?

WHAT CAN BE INFERRED FROM SUM RULES?

IF a single sharp resonance is observed (as in expt.) then:

$$\omega_{res}^2 = \gamma^2 H_{ext}^2 + \omega_0^2$$

$$\omega_0^2 = \gamma^2 \chi^{-1} \partial^2 \langle H_D \rangle / \partial \theta^2$$

nuclear dipole energy

angle of simultaneous rot^n. of all spins

But $\partial^2 \langle H_D \rangle / \partial\theta^2 \sim \langle H_D \rangle$:

So, exptl. value of $\omega_o^2\,(T) \Rightarrow$

$\langle H_D \rangle\,(T) \sim K(1 - T/T_A)$, $K \sim 10^{-3}$ ergs/cm^3

HOW CAN THIS BE?

$$\left\{ \begin{array}{lll} \uparrow & (\text{"bad"}) & \uparrow \\ \rightarrow & (\text{"good"}) & \rightarrow \end{array} \right.$$

$$\Delta E \lesssim \frac{\mu_o \mu_n^2}{r_o^3} \sim 10^{-7}\,\mathrm{K} \ll k_B T$$

So, prima facie, preference for "good" orientation over "bad" is at most

$\sim \Delta E / k_B T \sim 10^{-4}$ [actually, $\sim \Delta E / k_B T_F \sim 10^{-7}$]

$\Rightarrow$ expectation value of dipole energy much too small!

SPONTANEOUSLY BROKEN SPIN-ORBIT SYMMETRY

Ferromagnetic analogy:

FERROMAGNET

$$\hat{H} = \hat{H}_o + \hat{H}_z$$

↑ invariant under simult. rotation of all spins

extl. field

$$\hat{H}_z = -\mu_B \mathcal{H} \sum_i S_{zi}$$

breaks spin-rot.ⁿ symmetry

Paramagnetic phase ($T > T_c$): spins behave independently, kT competes with $\mu_B \mathcal{H} \Rightarrow$ polarization $\sim \mu_B \mathcal{H}/kT \ll 1 \Rightarrow$ $\langle H_z \rangle \sim N(\mu_B \mathcal{H})^2/kT$

Ferromagnetic phase ($T < T_c$): $\hat{H}_o$ forces all spins to lie parallel $\Rightarrow k_B T$ competes with $N\mu_B\mathcal{H}$ $\Rightarrow \langle S_z \rangle \sim 1 \Rightarrow \langle H_z \rangle \sim N\mu_B \mathcal{H}$

LIQUID ^{3}HE

$$\hat{H} = \hat{H}_o + \hat{H}_D$$

↑ invariant under relative rotation of spin + orbital coordinate systems

$$\hat{H}_D = g_D \sum_{ij} \left(\frac{\underline{\sigma}_i \cdot \underline{\sigma}_j - 3\underline{\sigma}_i \cdot \hat{\underline{r}}_{ij}\, \underline{\sigma}_j \cdot \hat{\underline{r}}_{ij}}{(r_{ij}^3 / r_o^3)} \right), \qquad g_D \equiv \mu_o \mu_n^2 / r_o^3$$

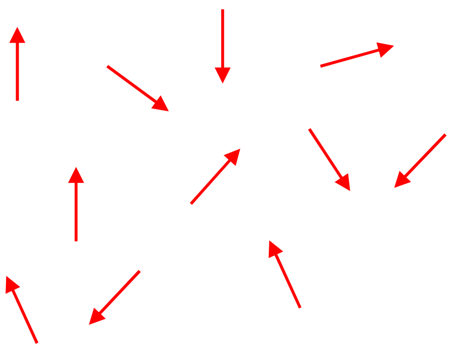
breaks relative spin-orbit rot.ⁿ symmetry

Normal phase ($T > T_A$): pairs of spins behave independently $\Rightarrow$ polarization $\sim g_D/kT \ll 1 \Rightarrow$ $\langle H_D \rangle \sim N g_D^2/kT$

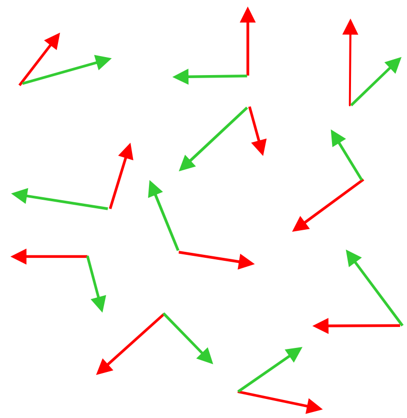
Ordered phase ($T < T_A$): $\hat{H}_o$ forces all pairs to behave similarly $\Rightarrow$ kT competes with Ng_D $\Rightarrow \langle H_D \rangle \sim Ng_D$ $\sim 10^{-3}$ ergs/cm^3 !

SBSOS: ORDERING MAY BE SUBTLE

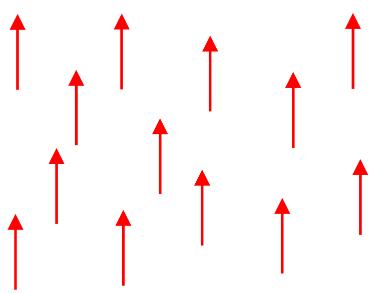
FERROMAGNET	LIQUID ^{3}HE



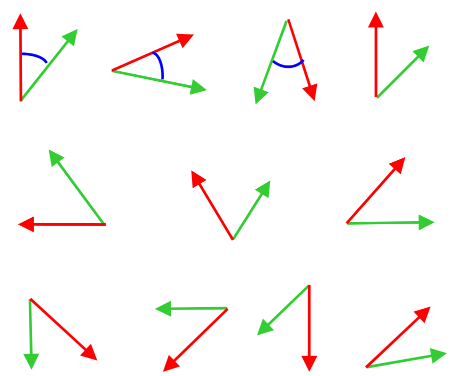
⇐ NORMAL PHASE ⇒



⇓ ⇓



⇐ ORDERED PHASE ⇒



( = total spin of pair

 = relative orbital ang. momentum)

$\langle \underset{\sim}{S} \rangle \neq 0$

$\langle \underset{\sim}{S} \rangle = \langle \underset{\sim}{L} \rangle = 0$

but $\langle \underset{\sim}{L} \times \underset{\sim}{S} \rangle \neq 0$!

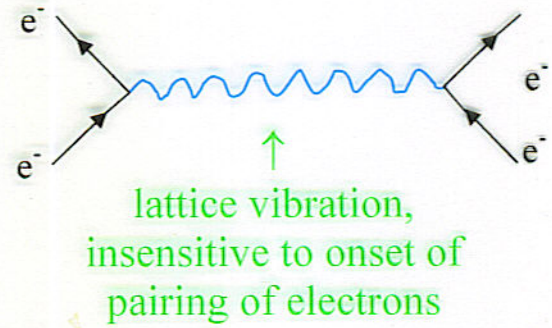
RESOLUTION OF THE PARADOX OF TWO NEW PHASES.

(Anderson & Brinkman, Phys. Rev. Letters **30**, 1108 (1973))

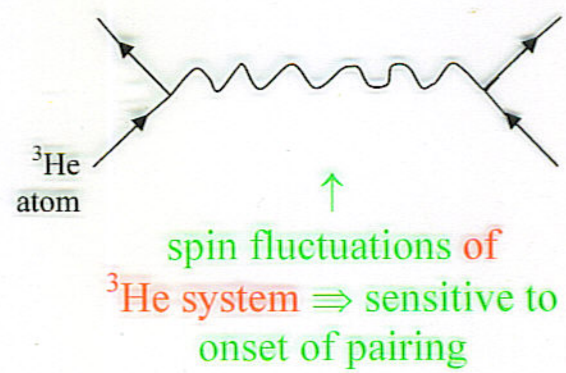
In BCS (weak-coupling) theory for $\ell = 1$, BW phase is always stable, independently of pressure and temperature.

Crucial difference between Cooper pairing in superconductors and ^{3}He:

Superconductor:



liquid ^{3}He:



⇒ "feedback" effects: Over most of the phase diagram, BW state stable as in BCS theory. But at high temperature and pressure, feedback effects uniquely favor ABM phase.

major qualitative leap beyond BCS!

MICROSCOPIC SPIN DYNAMICS (SCHEMATIC)

Basic variables:

(a) Total spin $\underset{\sim}{S}$

(b) Orientation $\underset{\sim}{\theta}$ of spin of Cooper pairs

$\Big\}$ $[S_i, \theta_j] = i\delta_{ij}$

$$\hat{H} = \hat{H}_o(\underset{\sim}{S}) + \hat{H}_D(\underset{\sim}{\theta})$$

↑

hydrodynamic (Born-Oppenheimer) approximation

Semiclassical equations of motion:

zero in eq^{m}! ↙ dipole torque ↙

$$\frac{d\underset{\sim}{\theta}}{dt} = \frac{\partial \langle \hat{H}_o \rangle}{\partial \underset{\sim}{S}} = \mathcal{H}_{ext} - \chi^{-1}\underset{\sim}{S}, \qquad \frac{d\underset{\sim}{S}}{dt} = \underset{\sim}{S} \times \mathcal{H}_{ext} - \frac{\partial \langle \hat{H}_D \rangle}{\partial \underset{\sim}{\theta}}$$

⇒ linear NMR behavior completely determined by eigenvalues of quantity

$$\Omega_{ij}^2 \equiv \partial^2 \langle H_D \rangle / \partial\theta_i\, \partial\theta_j$$

so, can "fingerprint" A and B phases by NMR!

ABM: single resonance line

axial: split resonance

BW: original BW state is $\underset{\sim}{L} = -\underset{\sim}{S}$, i.e. $J = 0$. But dipole torque rotates $\underset{\sim}{S}$ relative to $\underset{\sim}{L}$ by $\angle \cos^{-1}(-1/4) = 104°$ around axis $\hat{\underset{\sim}{\omega}}$ whose "best" choice is $\mathcal{H}_{ext}$.

Result: no shift in transverse resonance, but finite-frequency longitudinal resonance!
(also in ABM phase)

$\mathcal{H}_{ext}$ ↑ $\uparrow \mathcal{H}_{rf} \sim \cos \omega t$

CONCLUSION (by summer of 1973):

Both a priori stability considerations and NMR experimental data are consistent with hypothesis that both new phases are Cooper-paired ("superfluid") phases. Specifically,

A phase = ABM

B phase = BW

What is superfluid ^{3}He good for?

(a) most sophisticated physical system of which we can claim detailed quantitative understanding. E.g. textures, orientational dynamics, topological singularities…

(b) analogies with systems in particle physics, cosmology… (G. E. Volovik)

(c) studies of (some aspects of) turbulence

(d) Amplification of ultra-weak effects (cf NMR):
Example: P- (but not T-) violating effects of neutral current part of weak interaction:
For single elementary particle, any EDM $\underset{\sim}{d}$ must be of form

$\underset{\sim}{d} = \text{const. } \underset{\sim}{J}$ ← violates T as well as P.

But for ^{3}He – B, can form

$$d \sim \text{const. } \underset{\sim}{L} \times \underset{\sim}{S} \sim \text{const. } \hat{\underset{\sim}{\omega}}$$

↑

violates P but not T.

Effect is tiny for single pair, but since all pairs have same value of $\underset{\sim}{L} \times \underset{\sim}{S}$, is multiplied by factor of $\sim 10^{23}$ ⇒

macroscopic P-violating effect?