Evolution through maps

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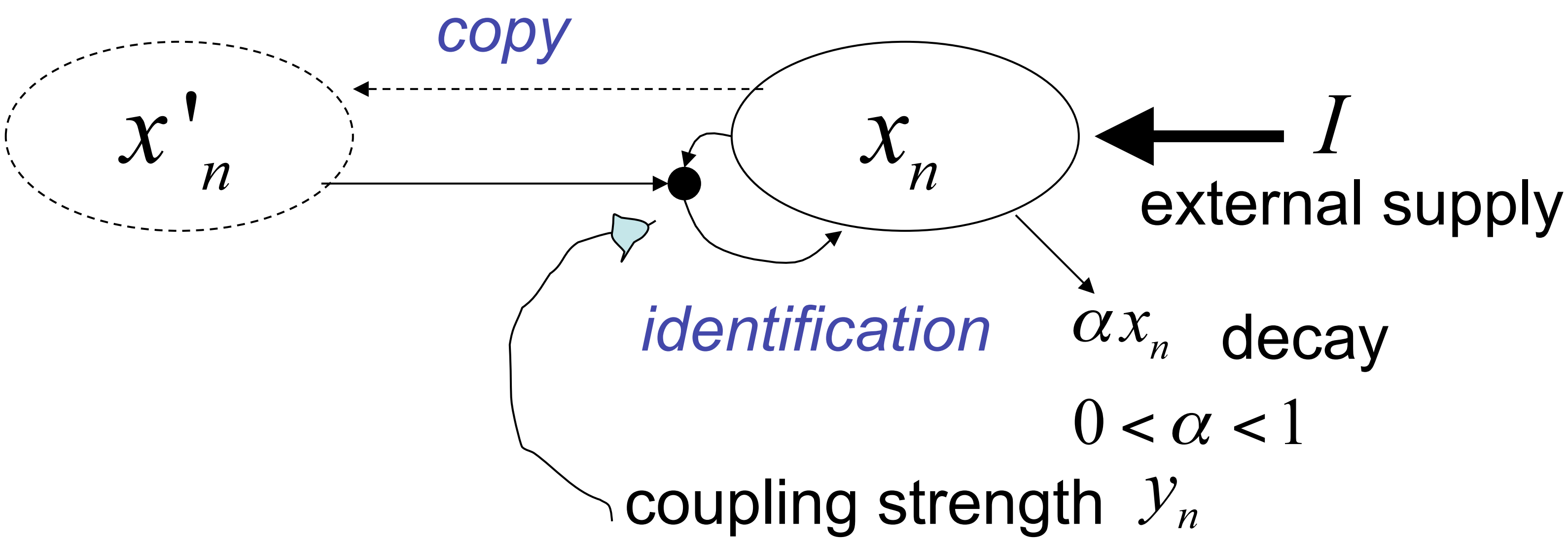
Similarity between *evolutionary* processes and *learning* processes:

gene (genotype)	⟷	synapse (synaptic states)
phenotype	⟷	neural states
selection pressure	⟷	selection of metric
landscape	⟷	neural manifold
selection rules	⟷	learning rules
individual	⟷	brain
mutation	⟷	synaptic & dendritic noise

1. The evolution by “copy-and-identify” process.



$$x_{n+1} = \alpha x_n + y_n x'_n x_n + I,$$
$$y_{n+1} = y_n + \beta x_n - \beta y_n x'_n x_n$$

$\alpha, \beta > 0$

y could be minus as well as positive.

We assume $x'_n = x_n$.

***Mirror neurons* in the brain**
(Giacomo Rizzolatti et al. 1996)

Embedded bifurcations

H. Kang and I. T., *Chaos* **19**, 033132(2009)1-12.

Brusselator (vector fields)

$$dx/dt = A - (1+B)x + x^2 y,$$

$$dy/dt = Bx - x^2 y \qquad (1)$$

Discrete Brusselator (direction fields)

$$x_{n+1} = x_n + \Delta t(A - (1+B)x_n + x_n^2 y_n),$$

$$y_{n+1} = y_n + \Delta t(Bx_n - x_n^2 y_n) \qquad (2)$$

$$f : R^2 \to R^2$$

$$f(x,y) = (f_1(x,y), f_2(x,y))$$

$$f_1(x,y) = a + (1 - \gamma - b)x + \gamma x^2 y$$

$$f_2(x,y) = y + bx - \gamma x^2 y$$

$$a = A\Delta t, \quad b = B\Delta t, \quad \text{and} \quad \gamma = \Delta t.$$

Regarding the variable y in Eq. (2) as a parameter, we obtain a one-dimensional map $g_y(x) = f_1(x, y)$, where

$$g_y(x) = \gamma y x^2 + (1 - \gamma - b)x + a. \quad (10)$$

$g_y(x)$ can be viewed as a *random logistic map.*

We can also prove that

(1) $y_{n+1} - y_n$ is the same order of discretization step γ .

(2) $\{y_n\}$ is a strictly monotone increase in the neighborhood of the bifurcations.

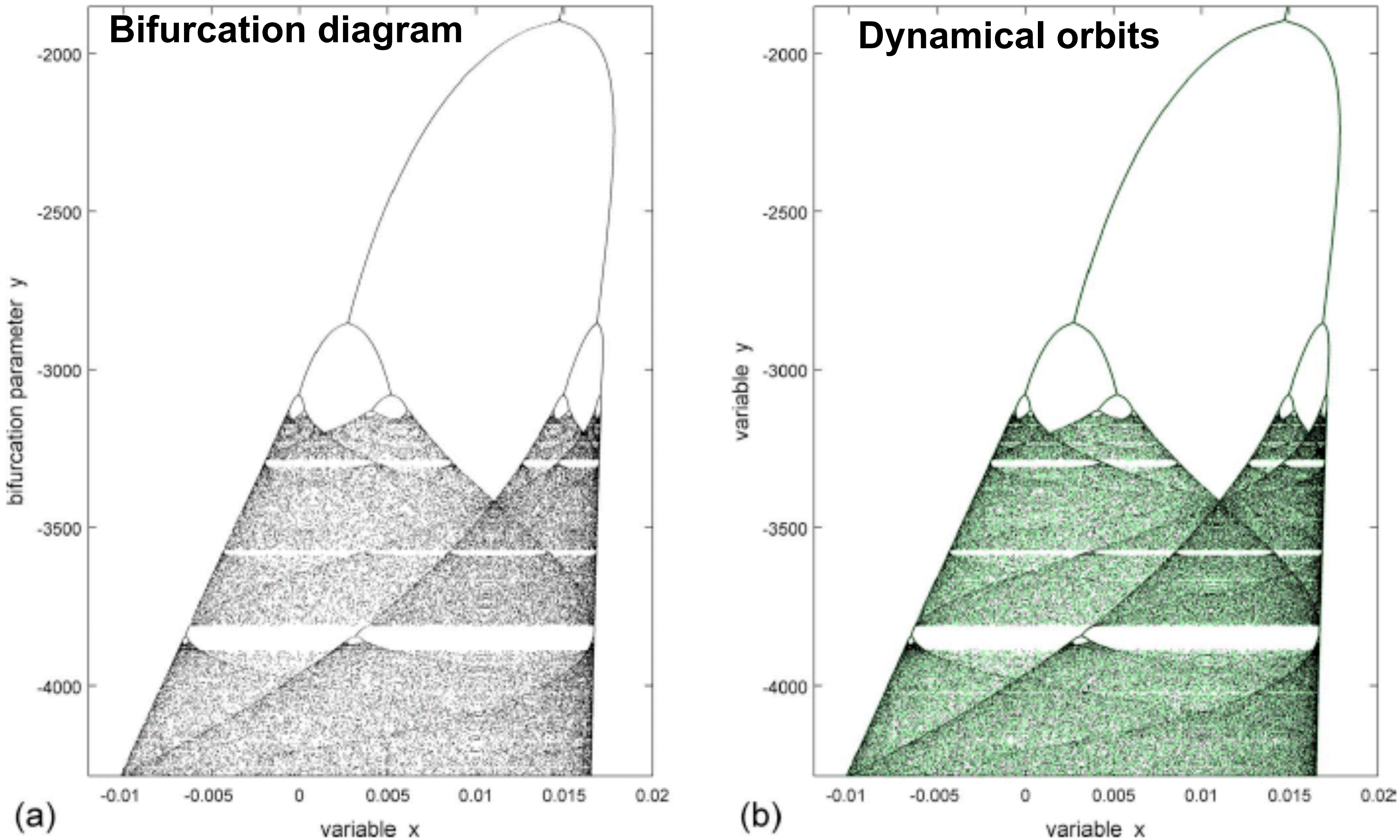


FIG. 1. (Color online) Comparison of a dynamical orbit of f to a bifurcation diagram of g_y at the same parameter values of $a = 0.015$, $b = 0.01$, and $\gamma = 0.035$. (a) The bifurcation diagram of a random logistic map. The bifurcation parameter y of $g_y(x)$ defined in Eq. (10) varies from -4284 to -1850. (b) A dynamical orbit of map f. This orbit originates in the point $(0, -4284)$, and about 160 000 iterations are displayed in the background of the bifurcation diagram in (a).

2. The evolution of mathematical functions

How could a neuron be evolved? **H. Watanabe, T. Ito, and I.T., 2009**

Varieties of functions:

$$z(t+1) = \tanh(\gamma_1(z(t)-\alpha_1)) - \omega \tanh(\gamma_2(z(t)-\alpha_2)) + J.$$

constant functions, monotonous functions (sigmoid functions), unimodal functions, and bimodal functions: a subset of polynomial.

After some appropriate selection algorithm (usually adopted in the literature), ***a threshold (step) function (extremely large*** γ ***)was selected*** among some wide classes of functions, thereby *excitable systems* are generated.

Selection pressure: maximum transmission of mutual information between input and each elementary individual map in the networks.

1. *<u>Chaotic inputs</u>*:

coupling constant

1-1. large: a selected function goes to zero.

1-2. intermediate: selected *step functions*,
by which pulse trains are propagated.

1-3. small: selected *step functions*,
by which oscillatory behaviors are propagated.

\+ noise: step functions more selected.

2. *<u>Periodic inputs</u>*: varieties of selected *periodic functions*
(*synchronized* with input), independent of
coupling constants.

Summary

*The discretized dynamical evolution shows some similarity to the natural selection.

*The presence of discrete time yields memory.

ex) Brower’s creation of number: interactions between consciousness and memory

I. Tsuda, Number created by the interaction between consciousness and memory: A mathematical basis for preafference, *Integrative Psychological and Behavioral Science* **42**(2), 153-156(2008).
Online publication: 25 May 2008; doi:10.1007/s12124-008-9062-y.

*A mathematical neuron (a threshold or a step function) evolves under the restriction of maximal transmission of information.

A unimodal map may be a model for cortical dynamics.

ref. X. Wang; M. Breakspear

Coupled excitatory (x) and inhibitory (y) neurons:

$$x(t+1) = f1(a\,x(t) - b\,y(t))$$
$$y(t+1) = f2(c\,x(t) - d\,y(t)),$$

where $fi\,(x)$ is a sigmoid function like $tanh(wi\,x)$.

Putting $u(t) = a\,x(t) - b\,y(t)$ and $v(t) = c\,x(t) - d\,y(t),$

$$u(t+1) = a\,f1(u(t)) - b\,f2(v(t))$$
$$v(t+1) = c\,f1(u(t)) - d\,f2(v(t)).$$

Assuming $a = c$ and $b = d$ for all t, we get 1-d map.

We can get ***bimodal*** 1-d ***chaotic*** map.(⇒see ***chaotic neuron*** by Aihara)

This map can be a ***unimodal chaotic map***,

restricted to the positive region, under some conditions, and,

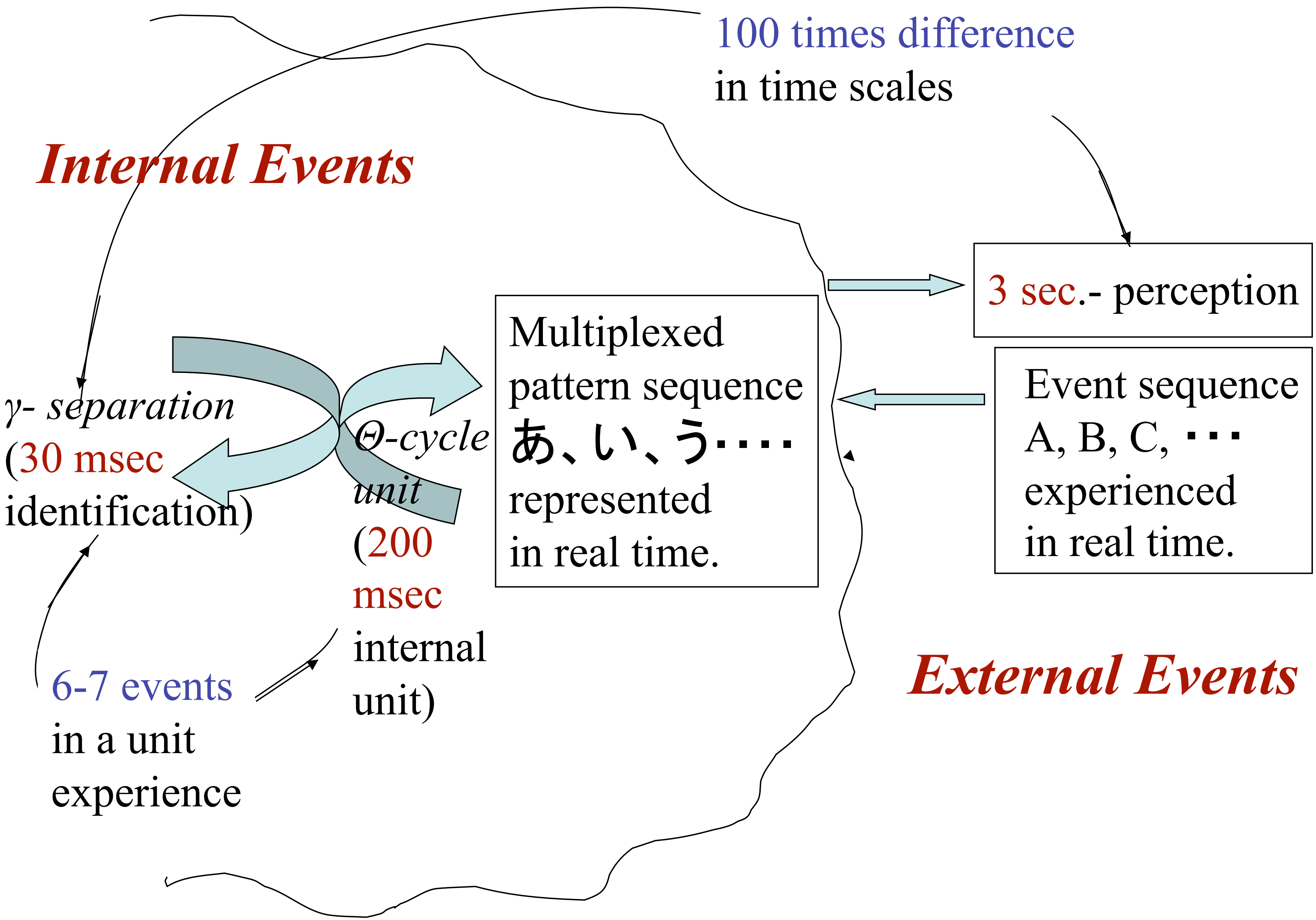
in particular, this chaotic map are ***excitable like BZ map***

rather than logistic map.

We also get a ***unimodal chaotic map*** like BZ map.

By the introduction of discrete time to logical inference, *contradiction* in classical logic disappears.

By taking continuous limit of time (description by differential equations), the solutions of classical logic recovers, associated with which *contradiction* reappears.


100 times difference
in time scales
Internal Events
3 sec.- perception
Multiplexed
pattern sequence
あ、い、う・・・・
represented
in real time.
Event sequence
A, B, C, ・・・
experienced
in real time.
γ- separation
(30 msec
identification)
Θ-cycle
unit
(200
msec
internal
unit)
External Events
6-7 events
in a unit
experience