A few problems on functional self-organization in the brain

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APCTP
Asia Pacific Center for Theoretical Physics

http://www.apctp.org

• Founded in 1996, located in Pohang, Korea
• 12 member countries, coop. with YITP, KEK, RIKEN, ISSP,…
• International cooperation, academic activities, young sci. training
• 3 M $/year, 40 activities, ~2,000 visitors
Outline

1. Introduction
2. Functional Self-organization in Cortical Development
3. Functional network formation in a STDP Neural Network Model
4. Functional Pathways in EEGs
5. Conclusion
Brain

• Neuron: Fundamental building blocks
  - slow (\sim 60Hz), nonlinear, complex response

• Massively interconnected
  1,000,000,000,000 neurons
  1,000,000,000,000,000 synapse

  Circuits: layers, feedback loops, ...
  “networks of networks”

• Highly complex & correlated responses

  \Rightarrow \text{Sensory information processing, movement coordination, learning, memory, cognition...}

• Activity dependent neural plasticity
  e.g. Learning and memory

Q: Structure \leftrightarrow Function?

Functional self-organization
Neural signal generation – Electrochemical


- Action potential: 1 msec, 0.1 V
- 0 & 1 signals of brain

- Weak stimulus
  - $E \geq 0$
  - $I = 1.4 \text{ nA}$

- Strong stimulus
  - $E \geq 0$
  - $I = 6.5 \text{ nA}$

- Ion conductances => currents
- Nonlinear voltage gating

Na-K pump
Modeling a neuron – Hodgkin–Huxley

Hodgkin-Huxley, 1963

Giant axon of a squid

Voltage clamp experiment, 1952

Reference electrode.

Current electrode

Recording electrode

Single-compartmental circuit model

⇒ H–H neuron “Circuits” – 4–d ODE
& variations of conduction–based models

Connection geometry: Global, local, sparse
Synaptic interactions: STDP, α, ...
External stimulus: dc, ac, noise

⇒ Dynamical System Models
for Nervous Dynamics
Studying self-organization in a brain hierarchy

- Nonlinear Time Series
  - Complex systems
  - Statistical physics
- Functional columns
  - Neural networks
- Nonlinear dynamics
  - Neuron
- Biophysics
  - Ion channel
  - Proteins

EEG/MEG pathways

Cortical models

Neural network models

Nonlinear dynamic models

Biophysical models
  - LTP, LTD, STDP
Functional Self–organization in visual maps

stimulus

Primary Visual cortex

Hubel (1989)
Orientation preference

visual stimulus

cortical neuron

neural response

Hubel, Wiesel, J. Physiol. (1962)
Optical imaging technique

Basis: neural activity ←→ haemodynamics

Bonhoeffer & Grinvald, J Neurosci (1993)
Orientation preference columns

Array of orientation preference (OP)
Orientation preference (OP) map

OP map of a tree shrew
(Bosking et al 1997)
Ocular dominance (OD) map in V

Ocular (left or right eye) dominance column

The structure of the primary visual cortex
Different visual map types in mammals

Macaque Monkey
- Stripe patterns in OD maps
- Strong OD segregation

Cat
- Island patterns in OD maps
- Intermediate OD segregation

Tree Shrew
- Weak OD segregation (no OD map)
- Strong orientation selectivity
A Biophysical model for visual map formation

\[ \frac{\partial \phi_i}{\partial t} = - \frac{\partial H}{\partial \phi_i} = -2\varepsilon \sum_j I(\mathbf{r}_i, \mathbf{r}_j) \sin(2\phi_i - 2\phi_j) \\
- 2\mu B_i \sin(2\phi_i - 2\phi'_i) \]

**LGN input**

2-dim layer of visual columns → Orientation preference

Coarse-grained picture at the functional column level

Neuron → functional column → area maps

Distance dependent interaction

**I(r)**: Mexican hat type

\[ I(r) = \frac{1}{2} \left( 1 - k \frac{r^2}{\sigma^2} \right) \exp(-r^2 / 2\sigma^2) \]

M.W. Cho & S. Kim, PRL, 2004
Analogy with a Spin Hamiltonian

Orientation Preference Ocular Dominance
$S_i = (\cos 2\phi, \sin 2\phi)$  $S_i = +1$ or $-1$

Maxican hat
Distance dependent

$H = - \sum_{i,j} J(\vec{r}_i, \vec{r}_j) S_i \cdot S_j - \sum h_i \cdot S_i$

X-Y model like

Ising model like

OP

OD
Time evolution of visual map over time

FIG. 2: Simulation results of the OP map using Eq. (1). Maps are generated with $\sigma^2 = 6$, $\varepsilon = 10^{-3}$, $\mu = 0$ (zero temperature), periodic boundary condition and an initially random state in $70 \times 70$ lattice.
OP & OD correlations in a spin model

\[ H = -\sum_{i,j} \left\{ J_{OP}(r_{ij})(S_i^{x} S_j^{x} + S_i^{y} S_j^{y}) + J_{OD}(r_{ij})S_i^{z} S_j^{z} \right\} \]

Red : \( S_x = 0 \) (OP, \( \phi = 0 \) or \( \pi/2 \))
Blue : \( S_y = 0 \) (OP, \( \phi = \pi/4 \) or \( 3\pi/4 \))
Black : \( S_z = 0 \) (OD, +/- boundary)

OP/OD correlation \( \leq \) synaptic input normalization \& competition

Theoretical predictions are consistent with experiments.
Analogy with vortices in magnetism

Generalized Model of OP+OD

\[ H = - \sum_{i,j} \left\{ J_{OP}(r_{ij})(S_i^x S_j^x + S_i^y S_j^y) + J_{OD}(r_{ij})S_i^z S_j^z \right\} \]

Exist bifurcations

- Bifurcation over k (inhibitory strength):
  homogeneous \( \Leftrightarrow \) spin waves (OP map)

- Bifurcation over \( \lambda \propto J_{OD}/J_{OP} \) (anisotropy)
  in-plane \( \Leftrightarrow \) out-of-plane vortices (OD map)

\( k < k_c \)
No OP maps

\( k > k_c \)
OP maps

OP pinwheels \( \Leftrightarrow \) in-plane vortices

OD beaded bands \( \Leftrightarrow \) out-of-plane vortices

\( k=0.2 \ \ \lambda=0.62 \)

Spin wave

\( k=0.2 \ \ \lambda=0.65 \)

Out-of-plane vortices

\( k=0.3 \ \ \lambda=0.62 \)

\( \lambda < \lambda_c \)
No OD maps

\( k=0.3 \ \ \lambda=0.65 \)

\( \lambda > \lambda_c \)
OD maps

* Anisotropic Heisenberg model

\[ H = -K \sum_{\langle i,j \rangle} \left\{ S_i^x S_j^x + S_i^y S_j^y + \lambda S_i^z S_j^z \right\} \]

M.W. Cho & S. Kim, PRB, 2004
Different OD map formation in a spin model

OD segregation strength

XY-like
Pure Heisenberg model

Instability of out-of-plane vortices
Ising-like

\( \lambda : \text{anisotropy between OP and OD columns} \quad \lambda \sim \frac{J_{OD}}{J_{OP}} \)

Wolf & Geisel, Nature 1998
Phase diagram of visual maps


\( \sigma_{\text{OD}}, \sigma_{\text{OP}} \): cooperation range in OP and OD columns

M.W. Cho & S. Kim, PRL, 2005
Functional maps in a brain

- Motor cortex
- Somatosensory cortex
- Primary visual cortex (V1)
- Primary auditory cortex (A1)
- Ocular dominance map
- Orientation preference map

Symmetry breaking
- in order parameter
- in space
Functional network formation in brain

\( fMRI \)

Equiluz et al, PRL64 2005

36x64x64 voxels
400 x 2.5s

• Measure \( V(x,t) \)
• Compute linear correlations
• Thresholded matrix
• Functional networks
Functional networks in a brain

Equiluz et al, PRL64 2005
f-MRI in humans

Robust for seven subjects at different tasks

Small-world

Scale-free

Previous studies

<table>
<thead>
<tr>
<th>$r_c$</th>
<th>$N$</th>
<th>$C$</th>
<th>$L$</th>
<th>$\langle k \rangle$</th>
<th>$\gamma$</th>
<th>$C_{\text{rand}}$</th>
<th>$L_{\text{rand}}$</th>
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<tr>
<td>0.6</td>
<td>31503</td>
<td>0.14</td>
<td>11.4</td>
<td>13.41</td>
<td>2.0</td>
<td>$4.3 \times 10^{-4}$</td>
<td>3.9</td>
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<tr>
<td>0.7</td>
<td>17174</td>
<td>0.13</td>
<td>12.9</td>
<td>6.29</td>
<td>2.1</td>
<td>$3.7 \times 10^{-4}$</td>
<td>5.3</td>
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<tr>
<td>0.8</td>
<td>4891</td>
<td>0.15</td>
<td>6.0</td>
<td>4.12</td>
<td>2.2</td>
<td>$8.9 \times 10^{-4}$</td>
<td>6.0</td>
</tr>
</tbody>
</table>

FIG. 2 (color online). Degree distribution for three values of the correlation threshold. The inset depicts the degree distribution for an equivalent randomly connected network.
Spike timing dependent plasticity (STDP)

LTP

STDP

LTD

Temporal order ↔ synaptic plasticity
Model of a STDP neural network

Dynamic Model of Neural Networks + Synaptic plasticity by STDP

**Neuron : FitzHugh–Nagumo model**

\[
e \dot{v} = I_{ion} + I_{syn} + I_{ext}
\]

\[
\dot{w} = v - w - b
\]

\[
I_{ion} = v(v - a)(1 - v) - w
\]

\[
I_{syn}(t) = \sum_{j \neq i} \left[ g_{ij}(t)(V - v_i(t)) + \bar{g}_{ij}(t)(\bar{V} - v_i(t)) \right]
\]

**Synapse : STDP**

\[
W(\Delta t) = \begin{cases} 
A_+ \exp\left(-\Delta t/\tau_+\right) & \text{if } \Delta t > 0 \\
-A_- \exp\left(\Delta t/\tau_-\right) & \text{if } \Delta t < 0 
\end{cases}
\]

\[
\Delta G_{ij} = G_{ij} \cdot W(\Delta t) \quad 0 < G_{ij} \leq G_{max}
\]

\[
\bar{G}_{ij}(t) = G_{inh}
\]

Initial all-to-all coupling

Q: How does the functional network emerge through self-organization?

Dynamics ⇔ Structure
- Activity dependent development
Complex network formation through self-organization

- Sparse, small world
- Scale-free properties

Fast, high coherence & fast, strong synchronization

$\Rightarrow$ Dynamically more effective and structurally more robust

C. W. Shin & S. Kim, PRE, 2006
Want to understand the correlation between the feedback integration in the frontal-posterior network with conscious information processing in brain.

Anesthesia experiments with human subjects

Analysis based on the Information Flow of EEG.

Cooperating with Dep. Anesthesiology, Hyundae Asan Hospital, Seoul, Korea

“Loss of consciousness (LOC)”

FeedForward (FF)

“Unconscious information”

Frontal

FeedBack (FB)

“Conscious information”

Sensory systems

Dynamics of consciousness in qualia space of EEG

**Research Goals:**

1. Development of a way to reconstruct the qualia space with EEG
2. Investigation of time evolution of conscious state.

Simultaneous activation of different local brain functions at a given time.

Qualia Space

- Axis 3 (F3-F4-C4)
- Axis 2 (F4-T3-T4-P4)
- Axis 1 (F3-T4-C3)

; Casually independent sub-clusters of EEGs during 0.5 second.

Brain: Frontier of complex systems

- Studies of functional self-organization in brain (NCSL)
  - Map formation in visual cortex: M.W. Cho & S. Kim, PRL ’04, PRL ’05, PRB ’05
  - Functional development of asymmetry and area differentiation, preprint, ‘07
  - Functional complex network formation by STDP: C.W. Shin & S. Kim, PRE ’06
  - Orientation tuning through synchronization: S.-G. Lee & S. Kim, PRE, ’05

- Brain: Paradigm for complex biological systems
  - Highly nonlinear neuron, and complex network structure
  - Self-organization and activity dependent neural plasticity

- Fertile interdisciplinary grounds for new quantitative methods
  Nonlinear Dynamics + Statistical Mechanics + Computations
  → Understanding biocomplexity of the nervous system & its role in brain function

Work in progress and challenges ahead!

* One of the 10 unsolved problems in physics!

*IOP, 1999
Complex networks

• Networks
  – node → network
  – regular, random, complex networks
  – Bionetworks, www, internet, airport, social relations

Q. Network structure vs functions?
  Eg. Robustness, efficiency

• Complex networks
  – Small World networks
    • Short path length: “six deg. of separation”
    • High clustering: A→B, B→C => A→C
  – Scale-Free networks
    • Degree distribution: Power-law \( p(k) \sim k^{-\gamma} \)
    \(<cf>\) random networks: Poisson distribution

Strogatz, Nature 410, 268 (2001)
Functional Structure by STDP

Globally coupled network

STDP

Functional Structure

Connection prob.

Clustering coeff. $C/C_{\text{rand}}$

Phase coherence

Fraction of connections between maximal connected neighbours

Degree of phase coherence among clusters
Small-world and Scale-free Network

Small-world properties

- clustering coefficient
  - $C = 0.23 \gg C_{rand} = 0.03$

- Average path length
  - $L = 3.19 \sim L_{rand} = 2.03$

Scale-free properties

- Degree distributions
  - $P_{in}(k) \sim k^{-\gamma_{in}}$
  - $P_{out}(k) \sim k^{-\gamma_{out}}$

$G_{max} = 0.2$, $G_{inh} = 0.02$
Self-organized Criticality

Fluctuation of synaptic coupling strength

- Functional structure lies in a self-organized critical state
- STDP events have no long-time correlation

Phase flips propagate in an avalanche manner

Quasi-steady state in network properties

<cf> sandpile
Diverse networks

WWW
Scale-free $\gamma=2.2$

Coauthorship
(arrow.org, SPIRES, NCSTRL)
Exponential cutoff

Power grid
exponential

Social (Mormon)
Gaussian
Synchrony in cat visual cortex

- Neurophysiological experiments on neuron pairs far apart
- Observation: Stimulus dependent, long-range synchronization
- Synchronization – same orientation columns
- Non-synchronization – different orientation columns

Engel et al, 1990
Orientation columns in visual cortex

Q: How do neurons become selective to a stimulus?

Orientation selectivity

Hubel & Wiesel, 1959

Elongated patterns of convergence from thalamic inputs

Observations:
LGN input weakly tuned. Fester et al, 1990
Orientation selectivity is sharp ~ 18°

Q: How is the weakly tuned input from LGN sharpened?

Orientation tuning <= Intracortical connections? relevance to synchronization?

Q: Why is the orientation tuning not dependent on contrast?

Contrast invariance
Orientation Column Model

Spiking neuron model at subcolumnar level

Synaptic connection model
- All-to-all excitatory within a column
- All-to-all excitatory between neighboring columns:
  Connection strength 15% smaller
- Inhibitory cells in the same column receive same inputs
- Global inhibitory to all excitatory cells
- Global inhibitory to all inhibitory cells

Neuron & synapse models
- Hodgkin-Huxley neurons
- α synaptic coupling (Jack, 1975)
- Correlated noisy current: Ornstein-Uhlenbeck

Orientation columns

Lee, Tanaka, Kim, 2004

- A column ➔ 12 excitatory cells + 3 inhibitory cells
- Angular difference: 12.85

225 neurons
Orientation tuning


- Orientation selectivity through synchronization \( \sim 17° \)
- Contrast invariance

- Quantifiers
  - Averaged firing rate of all columnar cells
  - PHPS (peak height of power spectrum) for synchrony

=> Robust

- Noise
  - Type I and Type II neurons (IF neurons)
  - Different synapses (e.g., Kinetic synapse model)
  - Size independence (# of neurons/# of columns)